Case 28

The minimal zero supports are given by (1, 2, 3), (1, 2, 4), (1, 3, 5), (2, 4, 5), (3, 4, 5), (2, 3, 6)

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & cos(\phi_2+\phi_3) & cos(\phi_1+\phi_4) & b_1 \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & -cos(\phi_3) & cos(\phi_3+\phi_5) & -cos(\phi_1+\phi_2-\phi_6) \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & cos(\phi_4+\phi_5) & -cos(\phi_4) & -cos(\phi_6) \\ cos(\phi_2+\phi_3) & -cos(\phi_3) & cos(\phi_4+\phi_5) & 1 & -cos(\phi_5) & b_2 \\ cos(\phi_1+\phi_4) & cos(\phi_3+\phi_5) & -cos(\phi_4) & -cos(\phi_5) & 1 & b_3 \\ b_1 & -cos(\phi_1+\phi_2-\phi_6) & -cos(\phi_6) & b_2 & b_3 & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1, u_2, u_3, u_4, u_5, u_6, u_7) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_3) \\ sin(\phi_2+\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_4) \\ 0 \\ sin(\phi_1+\phi_4) \\ 0 \\ sin(\phi_1) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_5) \\ 0 \\ sin(\phi_3+\phi_5) \\ sin(\phi_3) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_5) \\ sin(\phi_4) \\ sin(\phi_4+\phi_5) \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ sin(\phi_6) \\ sin(\phi_1+\phi_2-\phi \\ 0 \\ 0 \\ sin(\phi_1+\phi_2) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) + sin(\phi_2)cos(\phi_4+\phi_5) \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_1)cos(\phi_3+\phi_5) \\ b_1 sin(\phi_1+\phi_2) - sin(\phi_1+\phi_2)cos(\phi_1-\phi_6) \end{pmatrix} => \begin{cases} cos(\phi_1+\phi_2+\phi_3) \geq -cos(\phi_4+\phi_5) \\ cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_3+\phi_5) \\ b_1 \geq cos(\phi_1-\phi_6) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_3) + sin(\phi_2)cos(\phi_4+\phi_5) \\ 0 \\ sin(\phi_3)cos(\phi_1+\phi_4) + sin(\phi_3)cos(\phi_2+\phi_3+\phi_5) \\ b_2 sin(\phi_2) + b_1 sin(\phi_3) - sin(\phi_2+\phi_3)cos(\phi_1+\phi_2-\phi_6) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_2+\phi_3) \geq -cos(\phi_4+\phi_5) \\ cos(\phi_1+\phi_4) \geq -cos(\phi_2+\phi_3+\phi_5) \\ b_2 sin(\phi_2) + b_1 sin(\phi_3) - sin(\phi_2+\phi_3)cos(\phi_1+\phi_2-\phi_6) \geq 0 \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ sin(\phi_1)cos(\phi_1+\phi_2+\phi_4) + sin(\phi_1)cos(\phi_3+\phi_5) \\ 0 \\ sin(\phi_4)cos(\phi_2+\phi_3) + sin(\phi_4)cos(\phi_1+\phi_4+\phi_5) \\ 0 \\ b_3 sin(\phi_1) + b_1 sin(\phi_4) - sin(\phi_1+\phi_4)cos(\phi_6) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_2+\phi_4) \geq -cos(\phi_3+\phi_5) \\ cos(\phi_2+\phi_3) \geq -cos(\phi_1+\phi_4+\phi_5) \\ b_3 sin(\phi_1) + b_1 sin(\phi_4) - sin(\phi_1+\phi_4)cos(\phi_6) \geq 0 \end{cases}$$

4. $$Au_4 = \begin{pmatrix} sin(\phi_3)cos(\phi_1+\phi_4) + sin(\phi_3)cos(\phi_2+\phi_3+\phi_5) \\ 0 \\ sin(\phi_5)cos(\phi_3+\phi_4+\phi_5) + sin(\phi_5)cos(\phi_1+\phi_2) \\ 0 \\ 0 \\ b_3 sin(\phi_3) + b_2 sin(\phi_3+\phi_5) - sin(\phi_5)cos(\phi_1+\phi_2-\phi_6) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_4) \geq -cos(\phi_2+\phi_3+\phi_5) \\ cos(\phi_3+\phi_4+\phi_5) \geq -cos(\phi_1+\phi_2) \\ b_3 sin(\phi_3) + b_2 sin(\phi_3+\phi_5) - sin(\phi_5)cos(\phi_1+\phi_2-\phi_6) \geq 0 \end{cases}$$

5. $Au_5 = \begin{pmatrix} sin(\phi_4)cos(\phi_2+\phi_3) + sin(\phi_4)cos(\phi_1+\phi_4+\phi_5) \\ sin(\phi_5)cos(\phi_3+\phi_4+\phi_5) + sin(\phi_5)cos(\phi_1+\phi_2) \\ 0 \\ 0 \\ 0 \\ b_2 sin(\phi_4) + b_3 sin(\phi_4+\phi_5) - sin(\phi_5)cos(\phi_6) \end{pmatrix} =>$

$$\begin{cases} cos(\phi_2+\phi_3) \geq -cos(\phi_1+\phi_4+\phi_5) \\ cos(\phi_3+\phi_4+\phi_5) \geq -cos(\phi_1+\phi_2) \\ b_2 sin(\phi_4) + b_3 sin(\phi_4+\phi_5) - sin(\phi_5)cos(\phi_6) \geq 0 \end{cases}$$

6. $Au_6 = \begin{pmatrix} b_1 sin(\phi_1+\phi_2) - sin(\phi_1+\phi_2)cos(\phi_1-\phi_6) \\ 0 \\ 0 \\ b_2 sin(\phi_1+\phi_2) - sin(\phi_6)cos(\phi_3) + sin(\phi_1+\phi_2-\phi_6)cos(\phi_4+\phi_5) \\ b_3 sin(\phi_1+\phi_2) + sin(\phi_6)cos(\phi_3+\phi_5) - sin(\phi_1+\phi_2-\phi_6)cos(\phi_4) \\ 0 \end{pmatrix} =>$

$$\begin{cases} b_1 \geq cos(\phi_1-\phi_6) \\ b_2 sin(\phi_1+\phi_2) - sin(\phi_6)cos(\phi_3) + sin(\phi_1+\phi_2-\phi_6)cos(\phi_4+\phi_5) \geq 0 \\ b_3 sin(\phi_1+\phi_2) + sin(\phi_6)cos(\phi_3+\phi_5) - sin(\phi_1+\phi_2-\phi_6)cos(\phi_4) \geq 0 \end{cases}$$

First 5 minimal zeros are cyclic=> inequalities on ϕ_i :

$$\begin{cases} \phi_1+\phi_2+\phi_3+\phi_4+\phi_5 < \pi \\ \phi_1+\phi_2 > \phi_6 \end{cases}$$

Copositivity:

Sets $(1,3,4), (2,3,4), (1,2,5), (2,3,5), (1,4,5), (1,2,6), (1,3,6), (1,4,6), (2,4,6), (3,4,6)$, $(1,3,4,6), (1,5,6), (2,5,6), (1,2,5,6), (3,5,6), (4,5,6), (1,4,5,6)$ have to be considered.

As hold above conditions on ϕ_i, then necessary inequalities for sets $(1,3,4), (2,3,4), (1,2,5), (2,3,5), (1,4,5)$ are performed.

Consider $I = (2,4,6) : b_2 = cos(\phi), \phi_3+\phi_1+\phi_2-\phi_6 \geq \phi => b_2 = cos(\phi) \geq cos(\phi_3+\phi_1+\phi_2-\phi_6)$

If b_2 is determined from inequality 6.2 $b_2 = \frac{sin(\phi_6)cos(\phi_3) - sin(\phi_1+\phi_2-\phi_6)cos(\phi_4+\phi_5)}{sin(\phi_1+\phi_2)} \geq cos(\phi_3+\phi_1+\phi_2-\phi_6) =>$ $-sin(\phi_1+\phi_2-\phi_6)(cos(\phi_4+\phi_5)+cos(\phi_3+\phi_1+\phi_2)) \geq 0 => \phi_3+\phi_1+\phi_2+\phi_4+\phi_5 \geq \pi =>$ performs only if it's equation. Doesn't hold for our domain of ϕ_i.

If b_2 is determined from inequality 2.3 $b_2 = \frac{-sin(\phi_3)b_1 + sin(\phi_2+\phi_3)cos(\phi_1+\phi_2-\phi_6)}{sin(\phi_2)} \geq cos(\phi_3+\phi_1+\phi_2-\phi_6) =>$ $-sin(\phi_3)(b_1 - cos(\phi_1-\phi_6)) \geq 0 => b_1 \leq cos(\phi_1-\phi_6)$, but from inequality 1.3 $b_1 \geq cos(\phi_1-\phi_6) =>$ performs only if it's equation.

If b_2 is determined from inequality 4.3 $b_2 = \frac{sin(\phi_5)cos(\phi_1+\phi_2-\phi_6) - b_3 cos(\phi_3)}{sin(\phi_3+\phi_5)} \geq cos(\phi_3+\phi_1+\phi_2-\phi_6) => b_3 \leq$ $-cos(\phi_1+\phi_2+\phi_3+\phi_5-\phi_6)$, but from 6.3. follows $-cos(\phi_1+\phi_2+\phi_3+\phi_5-\phi_6) \geq b_3 \geq \frac{cos(\phi_4)sin(\phi_1+\phi_2-\phi_6) - sin(\phi_6)cos(\phi_3+\phi_5)}{sin(\phi_1+\phi_2)} =>$ $(-cos(\phi_1+\phi_2+\phi_3+\phi_5) - cos(\phi_4))sin(\phi_1+\phi_2-\phi_6) \geq 0 => \phi_3+\phi_1+\phi_2+\phi_4+\phi_5 \geq \pi =>$ performs only if it's equation. Doesn't hold for our domain of ϕ_i.

If b_2 is determined from inequality 5.3 $b_2 = \frac{sin(\phi_5)cos(\phi_6) - sin(\phi_4+\phi_5)b_3}{sin(\phi_4)} \geq cos(\phi_3+\phi_1+\phi_2-\phi_6)$ and it can be derived from initial inequalities, that $-cos(\phi_1+\phi_2+\phi_3-\phi_6) \leq cos(\phi_4+\phi_5+\phi_6)$ $sin(\phi_4)cos(\phi_4+\phi_5+\phi_6) + sin(\phi_5)cos(\phi_6) - sin(\phi_4+\phi_5)b_3 \geq 0 => b_3 \leq cos(\phi_4+\phi_6)$, but from copositivity of submatrix (3,5,6) follows, that $b_3 \geq cos(\phi_4+\phi_6) =>$ performs only if it's equation.

As result, there appears additional minimal zero (2, 4, 6) in all possible cases for b_2. There is no copositivty.

Result

There aren't any copositive matrices with such minimal zeros set.

References

[1] Peter J.C. Dickinson. *A new certificate for copositivity. Optimization Online.*

[2] Peter J.C. Dickinson and Roland Hildebrand. *Considering copositivity locally.* J.Math. Anal.Appl., 437(2):1184–1195, 2016

.