Case 25

The minimal zero supports are given by (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 6), (3, 4, 6), (4, 5, 6)

$$A = \begin{pmatrix} 1 & -cos(\phi_2) & -cos(\phi_1) & cos(\phi_2+\phi_3) & cos(\phi_2+\phi_4) & cos(\phi_1+\phi_5) \\ -cos(\phi_2) & 1 & cos(\phi_1+\phi_2) & -cos(\phi_3) & -cos(\phi_4) & b_1 \\ -cos(\phi_1) & cos(\phi_1+\phi_2) & 1 & cos(\phi_5+\phi_6) & b_2 & -cos(\phi_5) \\ cos(\phi_2+\phi_3) & -cos(\phi_3) & cos(\phi_5+\phi_6) & 1 & cos(\phi_6+\phi_7) & -cos(\phi_6) \\ cos(\phi_2+\phi_4) & -cos(\phi_4) & b_2 & cos(\phi_6+\phi_7) & 1 & -cos(\phi_7) \\ cos(\phi_1+\phi_5) & b_1 & -cos(\phi_5) & -cos(\phi_6) & -cos(\phi_7) & 1 \end{pmatrix}$$

The zeros are given by the columns:

$$(u_1,u_2,u_3,u_4,u_5,u_6,u_7) = \begin{pmatrix} sin(\phi_1+\phi_2) \\ sin(\phi_1) \\ sin(\phi_2) \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_3) \\ sin(\phi_2+\phi_3) \\ 0 \\ sin(\phi_2) \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_4) \\ sin(\phi_2+\phi_4) \\ 0 \\ 0 \\ sin(\phi_2) \\ 0 \end{pmatrix} \begin{pmatrix} sin(\phi_5) \\ 0 \\ sin(\phi_1+\phi_5) \\ 0 \\ 0 \\ sin(\phi_1) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ sin(\phi_6) \\ sin(\phi_5) \\ 0 \\ sin(\phi_5+\phi_6) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ sin(\phi_7) \\ sin(\phi_6) \\ sin(\phi_6+\phi_7) \end{pmatrix}$$

1. $$Au_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_3)+sin(\phi_2)cos(\phi_5+\phi_6) \\ b_2 sin(\phi_2)+sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) \\ b_1 sin(\phi_1)+sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) \end{pmatrix} => \begin{cases} cos(\phi_1+\phi_2+\phi_3) \geq -cos(\phi_5+\phi_6) \\ b_2 \geq -cos(\phi_1+\phi_2+\phi_4) \\ b_1 \geq -cos(\phi_1+\phi_2+\phi_5) \end{cases}$$

2. $$Au_2 = \begin{pmatrix} 0 \\ 0 \\ sin(\phi_2)cos(\phi_1+\phi_2+\phi_3)+sin(\phi_2)cos(\phi_5+\phi_6) \\ 0 \\ -sin(\phi_2)cos(\phi_3-\phi_4)+sin(\phi_2)cos(\phi_6+\phi_7) \\ b_1 sin(\phi_2+\phi_3)+sin(\phi_3)cos(\phi_1+\phi_5)-sin(\phi_2)cos(\phi_6) \end{pmatrix}$$
$$=> \begin{cases} cos(\phi_1+\phi_2+\phi_3) \geq -cos(\phi_5+\phi_6) \\ cos(\phi_6+\phi_7) \geq cos(\phi_3-\phi_4) \\ b_1 sin(\phi_2+\phi_3)+sin(\phi_3)cos(\phi_1+\phi_5)-sin(\phi_2)cos(\phi_6) \geq 0 \end{cases}$$

3. $$Au_3 = \begin{pmatrix} 0 \\ 0 \\ b_2 sin(\phi_2)+sin(\phi_2)cos(\phi_1+\phi_2+\phi_4) \\ -sin(\phi_2)cos(\phi_3-\phi_4)+sin(\phi_2)cos(\phi_6+\phi_7) \\ 0 \\ b_1 sin(\phi_2+\phi_4)+sin(\phi_4)cos(\phi_1+\phi_5)-sin(\phi_2)cos(\phi_7) \end{pmatrix}$$
$$=> \begin{cases} b_2 \geq -cos(\phi_1+\phi_2+\phi_4) \\ cos(\phi_6+\phi_7) \geq cos(\phi_3-\phi_4) \\ b_1 sin(\phi_2+\phi_4)+sin(\phi_4)cos(\phi_1+\phi_5)-sin(\phi_2)cos(\phi_7) \geq 0 \end{cases}$$

4. $$Au_4 = \begin{pmatrix} 0 \\ b_1 sin(\phi_1)+sin(\phi_1)cos(\phi_1+\phi_2+\phi_5) \\ 0 \\ sin(\phi_5)cos(\phi_2+\phi_3)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ b_2 sin(\phi_1+\phi_5)+sin(\phi_5)cos(\phi_2+\phi_4)-sin(\phi_1)cos(\phi_7) \\ 0 \end{pmatrix}$$
$$=> \begin{cases} b_1 \geq -cos(\phi_1+\phi_2+\phi_5) \\ cos(\phi_2+\phi_3) \geq -cos(\phi_1+\phi_5+\phi_6) \\ b_2 sin(\phi_1+\phi_5)+sin(\phi_5)cos(\phi_2+\phi_4)-sin(\phi_1)cos(\phi_7) \geq 0 \end{cases}$$

5. $Au_5 = \begin{pmatrix} sin(\phi_5)cos(\phi_2+\phi_3)+sin(\phi_5)cos(\phi_1+\phi_5+\phi_6) \\ -sin(\phi_5)cos(\phi_3)+sin(\phi_6)cos(\phi_1+\phi_2)+b_1 sin(\phi_5+\phi_6) \\ 0 \\ 0 \\ b_2 sin(\phi_6)-sin(\phi_6)cos(\phi_5-\phi_7) \\ 0 \end{pmatrix} =>$

$$\begin{cases} cos(\phi_2+\phi_3) \geq -cos(\phi_1+\phi_5+\phi_6) \\ -sin(\phi_5)cos(\phi_3)+sin(\phi_6)cos(\phi_1+\phi_2)+b_1 sin(\phi_5+\phi_6) \geq 0 \\ b_2 \geq cos(\phi_5-\phi_7) \end{cases}$$

6. $Au_6 = \begin{pmatrix} sin(\phi_6)cos(\phi_2+\phi_4)+sin(\phi_7)cos(\phi_2+\phi_3)+sin(\phi_6+\phi_7)cos(\phi_1+\phi_5) \\ -sin(\phi_6)cos(\phi_4)-sin(\phi_7)cos(\phi_3)+b_1 sin(\phi_6+\phi_7) \\ b_2 sin(\phi_6)-sin(\phi_6)cos(\phi_5-\phi_7) \\ 0 \\ 0 \\ 0 \end{pmatrix} =>$

$$\begin{cases} sin(\phi_6)cos(\phi_2+\phi_4)+sin(\phi_7)cos(\phi_2+\phi_3)+sin(\phi_6+\phi_7)cos(\phi_1+\phi_5) \geq 0 \\ -sin(\phi_6)cos(\phi_4)-sin(\phi_7)cos(\phi_3)+b_1 sin(\phi_6+\phi_7) \geq 0 \\ b_2 \geq cos(\phi_5-\phi_7) \end{cases}$$

Consider inequalities on ϕ_i :
from inequalities 1.1 and 1.2 follows $\phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \leq \pi$
from inequalities 2.2 follows $\phi_6+\phi_7 \leq |\phi_3-\phi_4|$

Copositivity:

Sets $(1,3,4), (2,3,4), (1,3,5), (2,3,5), (1,4,5), (2,4,5), (3,4,5), (1,3,4,5), (2,3,4,5), (1,2,6), (2,3,6), (1,4,6), (2,4,6), (1,5,6), (2,5,6), (3,5,6), (2,3,5,6)$ have to be considered.

1. $I = (1,3,4) : -\phi_1+\phi_2+\phi_3+\phi_5+\phi_6 \leq \pi$ holds strictly
2. $I = (2,3,4) : \phi_1+\phi_2-\phi_3+\phi_5+\phi_6 \leq \pi$ holds strictly
3. $I = (1,4,5) : \phi_4+2\phi_2+\phi_3+\phi_6+\phi_7 \leq 2\pi$ holds strictly
4. $I = (2,4,5) : \phi_6+\phi_7 \leq \phi_3+\phi_4$ holds strictly
5. $I = (1,4,6) : \phi_1+\phi_2+\phi_3+\phi_5-\phi_6 \leq \pi$ holds strictly
6. $I = (1,5,6) : \pi \geq -\phi_7+\phi_1+\phi_2+\phi_4+\phi_5$

Consider sets, that relates to b_2 :

1. $I = (2,5,6) : b_2 = cos(\phi) => b_2 \geq cos(\phi_4+\phi_7), \phi_4+\phi_7 < \pi$ In opposite case it holds true.
2. $I = (2,4,6) : b_2 = cos(\phi) => b_2 \geq cos(\phi_3+\phi_6)$

1. If $b_2 = \frac{sin(\phi_2)cos(\phi_6)-sin(\phi_3)cos(\phi_1+\phi_5)}{sin(\phi_2+\phi_3)} \geq cos(\phi_3+\phi_6)$ from inequality 2.3
   $-cos(\phi_2+\phi_3+\phi_6) \geq cos(\phi_1+\phi_5) => \phi_1+\phi_5+\phi_2+\phi_3+\phi_6 \geq \pi$ - holds only if it's an equation
2. If $b_2 = \frac{sin(\phi_5)cos(\phi_3)-sin(\phi_6)cos(\phi_1+\phi_2)}{sin(\phi_5+\phi_6)} \geq cos(\phi_3+\phi_6)$ from inequality 5.2
   $-cos(\phi_3+\phi_5+\phi_6) \geq cos(\phi_1+\phi_2) => \phi_1+\phi_5+\phi_2+\phi_3+\phi_6 \geq \pi$ - holds only if it's an equation
3. If $\phi_4 \geq \phi_3+\phi_6+\phi_7 : b_2 = \frac{sin(\phi_6)cos(\phi_4)+sin(\phi_7)cos(\phi_3)}{sin(\phi_6+\phi_7)} \geq cos(\phi_3+\phi_6)$ from inequality 6.2
   $cos(\phi_4) \geq cos(\phi_3+\phi_6+\phi_7) => \phi_4 \leq \phi_3+\phi_6+\phi_7$ - holds only if it's an equation

   If $\phi_3 \geq \phi_4+\phi_6+\phi_7 => cos(\phi_3+\phi_6) \leq cos(\phi_4+\phi_7) => b_2 = \frac{sin(\phi_6)cos(\phi_4)+sin(\phi_7)cos(\phi_3)}{sin(\phi_6+\phi_7)} \geq cos(\phi_4+\phi_7)$
   $cos(\phi_3) \geq cos(\phi_4+\phi_6+\phi_7) => \phi_3 \leq \phi_4+\phi_6+\phi_7$ - holds only if it's an equation

4. If $\phi_3 \geq \phi_4+\phi_6+\phi_7 => cos(\phi_3+\phi_6) \leq cos(\phi_4+\phi_7) => b_2 = \frac{-sin(\phi_4)cos(\phi_1+\phi_5)+sin(\phi_2)cos(\phi_7)}{sin(\phi_2+\phi_4)} \geq cos(\phi_4+\phi_7)$ from inequality 3.3
   $=> \phi_1 + \phi_2 + \phi_4 + \phi_5 + \phi_7 \geq \pi$ - doesn't hold
   If $\phi_4 \geq \phi_3 + \phi_6 + \phi_7 : b_2 = \frac{-sin(\phi_4)cos(\phi_1+\phi_5)+sin(\phi_2)cos(\phi_7)}{sin(\phi_2+\phi_4)} \geq cos(\phi_3 + \phi_6)$
   From copositivity of set (1, 5, 6) we need: $\pi-(\phi_1+\phi_5) \geq \phi_2+\phi_4-\phi_7 => -cos(\phi_1+\phi_5) \leq cos(\phi_2+\phi_4-\phi_7)$
   $sin(\phi_4)cos(\phi_2+\phi_4-\phi_7)+sin(\phi_2)cos(\phi_7)-sin(\phi_2+\phi_4)cos(\phi_3+\phi_6) \geq 0 => cos(\phi_4-\phi_7) \geq cos(\phi_3+\phi_6)$
   $\phi_4 - \phi_7 \leq \phi_3 + \phi_6$ - holds only if it's an equation

There is no copositityity.

Result

There aren't any copositive matrices with such minimal zeros set.

References

[1] Peter J.C. Dickinson. *A new certificate for copositivity. Optimization Online.*

[2] Peter J.C. Dickinson and Roland Hildebrand. *Considering copositivity locally.* J.Math. Anal.Appl., 437(2):1184–1195, 2016

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