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CLASSIFICATION OF PHASE PORTRAITS OF OPTIMAL SYNTHESIS

specialization: differential equations

T h e s i s   s u m m a r y

Moscow — 2000

This thesis was written at the Chair of Optimal Control of the Department of Mechanics and Mathematics of Moscow State University

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GENERAL DESCRIPTION

Introduction. In mathematics it is often convenient to consider whole classes of objects instead of single objects. This point of view is justified by the existence of transformation groups, which carry different objects into one another and define equivalence relations between them. If we are able to find a simple canonical object in each orbit and to investigate the properties of these concrete objects, then we can describe also any other object on the corresponding orbits. The search for canonical forms and their classification is a problem that is very often encountered in mathematics.

The group of transformations in optimal control is the *feedback group.* Finding canonical forms with respect to the feedback group and their classification is a problem that has been investigated by many authors.

Brunovský [1] has classified the class of linear systems with respect to the feedback group and introduced a corresponding canonical form, the Brunovský form. Later this problem was investigated by many authors (see e.g. [2],[3]), and was reduced to a purely algebraic finite-dimensional problem.

The feedback group of nonlinear systems is infinite-dimensional, which makes a classification considerably more difficult (see e.g. [4] or the survey [5]). A related problem is the stabilization of equilibrium points by dynamic feedback (see e.g. the survey [6]). An important problem is the identification of the orbits of linear systems, i.e. the description of those systems which can be linearized by a feedback transformation. A local linearizability criterion is given by an infinite number of algebraic conditions on the Lie brackets of the involved vector fields (see e.g. [7]).

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[2] Brockett R.W. *Some geometric questions in the theory of linear systems.* IEEE Trans. Aut. Contr., AC-21 (1976), no.4, 449-455.

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[4] Tchoń K. *On normal forms of affine systems under feedback.* New Trends in Nonlinear Control Theory. Proc. Conf., Nantes 1988, Lecture notes in Control and Information Sciences 122, Springer, Berlin, 1989, 23-32.

[5] Jakubczyk B. *Equivalence and invariants of nonlinear control systems.* Nonlinear controllability and optimal control, 177-218. Monographs and Textbooks Pure Appl. Math. 133, Dekker, NY, 1990.

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In order to avoid checking an infinite number of conditions the notion of *approximate feedback group* was introduced. This is a transformation group which takes nonlinear systems into one another while neglecting terms of orders higher than a specified number. Equivalence criteria with respect to the approximate feedback group were found by Krener (see e.g. survey [8]). These criteria amount to a finite number of conditions on the jets of order equal to the order of the feedback group.

The action of the approximate feedback group on the space of jets was investigated by Tchoń (see e.g. [9]). He showed that it boils down to the action of a finite-dimensional Lie group. Kang and Krener investigated normal forms of systems with respect to the action on second order jets [10].

If the set of admissible controls is a polyhedron, then the set of admissible phase velocities is the convex hull of a finite number of vector fields. Hence the subgroup of the feedback group which preserves the set of admissible controls acts in the same way as the group of diffeomorphisms acts on finite families of vector fields. This issue was investigated by many authors. Bogdanov (see [11],[12]) investigated the action of diffeomorphism groups of different smoothness classes on a vector field in the plane. A thorough classification of pairs of vector fields in the plane was done by Davydov in [13]. The case of more than two vector fields is considered as well. A related problem is the classification of distributions, which was investigated e.g. in [14].

Many authors obtained classifications of different classes of optimal control systems in the plane. Baitman [15] investigated time-optimal control systems. Recently Bressan and Piccoli obtained a complete topological classification of optimal syntheses of such systems in the vicinity of a singular point of the drift vector field [16]. Jacubczyk and Respondek classified planar systems with unrestricted control with respect to the action of the feedback group and the weak feedback group [17]. The weak feedback group includes diffeomorphisms

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[15]Baitman M.M. *Switching curves on the plane.* Differential equations 14 (1978), no.9, 1539-1551 (in Russian).

[16]Bressan A., Piccoli B. *Structural stability for time-optimal planar syntheses.* Dynam. Contin. Discr. Impuls. Syst. 3 (1997), no.3, 335-371.

[17]Jakubczyk B., Respondek W. *Feedback classification of analytic control systems in the plane.* Analysis of

of the phase space and feedback transformations as well as monotonic changes of the time scale.

Such systems occur e.g. in population dynamics. The uncontrolled system is of Volterra-Lottka type, while the control is unilateral. Systems with drift vector fields containing singularities of focus type and with integral cost functionals were considered in [18],[19]. In [18] we considered a mathematical pendulum which had to be brought to rest by application of a unilateral bounded force, while the mean-squared deviation from the equilibrium had to be minimized. In this work we considered Volterra-Lottka systems as well. In [19] systems from population dynamics with time-dependent cost were considered. In this thesis we investigate generic autonomous systems.

In §2.3 of this thesis we prove an assertion on the structure of a homeomorphism in the neighbourhood of a hyperbolic fixed point. Although it plays an auxiliary role in this thesis, it is itself an independent result.

In the literature there are basically three approaches to the problem of describing hyperbolic fixed points of diffeomorphisms. A classical result due to Poincaré [20] establishes sufficient linearizability conditions on the eigenvalues of the linear part of an analytic system of ODE. His idea was to compute the Taylor series expansion of the linearizing diffeomorphism in dependence of the Taylor series expansion of the vector field on the right-hand side of the ODE. The above-mentioned conditions guarantee convergence of this series. As a special case Poincaré proved that a planar system of ODE has a stable and an unstable invariant submanifold in the neighbourhood of a hyperbolic singular point, and that these submanifolds are analytic.

Hadamard [21] proposed another approach. He considered the stable and the unstable submanifolds as graphs of functions, which are fixed points with respect to the graph transformation induced by the diffeomorphism in question. It appears that this graph transformation is contracting in a suitably chosen metric space. This allows to apply Banach's fixed point theorem to prove the existence of these submanifolds. On one hand, Hadamard's approach allowed to treat also diffeomorphisms of finite smoothness classes. On the other hand, it only guaranteed that the submanifolds are Lipschitzian.

A third approach was proposed by Perron [22]. He constructed an functional

controlled dynamical systems (Lyon, 1990), 263-273. Progr. Systems Control Theory 8, Birkhäuser Boston, Boston, MA, 1991.

[18] Zelikin M.I., Zelikina L.F., Hildebrand R. *Asymptotics of optimal synthesis for one class of extremal problems.* Proceedings of Steklov Mathematical Institute, 233 (2001), 87-115.

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identity for the functions defining the invariant submanifolds.

In the thirties Petrovsky [23] proved that continuous differentiability of the systems's right-hand side implies continuous differentiability of the invariant submanifolds in the neighbourhood of a hyperbolic point.

In the fifties Sternberg [24] showed the existence of invariant submanifolds for homeomorphisms in the plane which are differerentiable in the hyperbolic singular point but only satisfying a certain Lipschitz condition elsewhere. For the case of diffeomorphisms he showed that the invariant submanifolds will be of the same smoothness class as the diffeomorphism. The proof of existence used the method of Hadamard, while the proof of smoothness used the method of Poincaré. Moreover, he developed a method [25] which allowed to reduce the investigation of a system of ODE to the case of a diffeomorphism.

In §2.3 of this thesis we consider homeomorphisms on the plane, which derive from a system of ODE with asymptotics $|\dot{x}| = O(|x| \ln |x|)$ as x approaches the origin. Thus the right-hand side of the system is not differentiable in the singular point. We show that certain conditions, which in some sense are hyperbolicity conditions on the singular point, imply the existence of invariant manifolds. Moreover, these manifolds possess the same smoothness class as the right-hand side of the system in a punctured neighbourhood of the singular point. this was accomplished using the method of Hadamard. We show also that if we can transform the homeomorphism to an analytic mapping by a (non-smooth) coordinate transformation of a certain kind, then the invariant manifolds will be analytic in the original system of coordinates. To prove this we used the method of Poincaré.

Main results. The main results of the thesis include the following.

i) We solved the planar optimal control problem in the neighbourhood of a singularity of focus type for autonomous systems which are affine in a scalar bounded unilateral control.

ii) We described the transformation group for this class of systems, defined a canonical form for each orbit and classified these canonical forms with respect to the topological structure of the optimal synthesis.

iii) We found the critical parameter values where bifurcations of the system occur.

[23] Petrovsky I.G. *On the behaviour of integral curves of a system of differential equations in the neighbourhood of a singular point.* Mat. Sb. (new series) 41 (1934)

[24] Sternberg S. *On the behaviour of invariant curves near a hyperbolic point of a surface transformation.* American Journal of Mathematics 77 (1955), 526-534

[25] Sternberg S. *Local contractions and a theorem of Poincaré.* American Journal of Mathematics 79 (1957), 809-824

iv) We proved a theorem on the existence and smoothness of invariant submanifolds of a homeomorphism in the neighbourhood of a degenerated hyperbolic fixed point.

Item i) helps also to clarify the occurrence of chattering in systems where the singular control assumes values on the boundary of the admissible set.

Structure. The thesis consists of an introduction and 4 chapters.

CONTENTS

In the **introduction** we give on overview over the literature and define the class of systems that are investigated in the thesis. We consider planar systems given by

$$\dot{x} = A(x) + B(x)u; \qquad x \in U \subset \mathbf{R}^2, \quad u \in [0,1], \quad \lim_{t\to+\infty} x(t) = \tilde{x}. \tag{1}$$

The control $u(t)$ is to be chosen as a measurable function in order to minimize the cost functional

$$J = \int_0^\infty (F(x) + uG(x))\,dt \;\to\; \min. \tag{2}$$

The point $\tilde{x}$ is the terminal manifold, while U is a neighbourhood of $\tilde{x}$. A, B are vector fields and F, G are scalar functions. We consider two cases, namely when these objects belong to the smoothness classes C^3 and C^ω, respectively. The drift vector field A is singular in $\tilde{x}$, $A(\tilde{x}) = 0$. The singularity is of focus or centre type, i.e. the Jacobian $\frac{\partial A}{\partial x}(\tilde{x})$ has complex conjugated eigenvalues. The vector field B is non-zero in the neighbourhood of $\tilde{x}$. The scalar function F and its gradient vanish at $\tilde{x}$. The function G also vanishes at $\tilde{x}$. The admissible trajectories $x(t)$ are supposed not to leave the neighbourhood U.

The first chapter deals with the symmetry group of the class of systems defined by (1),(2). We show that this group, let us denote it by $\mathbf{G}$, is generated by four subgroups Δ, Ξ, Λ, R. Here Δ is the group of diffeomorphisms in the plane. Ξ is the group of canonical (with respect to the Hamiltonian structure) transformations of the extended phase space which leave the projection on the phase space and the affine structure with respect to the control invariant. The subgroup Λ includes monotonic changes of the time scale, while R corresponds to multiplication of the cost function by positive constants.

Proposition: *Any element $\mathbf{g} \in \mathbf{G}$ possesses a unique decomposition $\mathbf{g} = D \circ \xi \circ \lambda \circ r$, where $D \in \Delta$, $\xi \in \Xi$, $\lambda \in \Lambda$, $r \in R$.*

The action of the group $\mathbf{G}$ on the jet space at $\tilde{x}$ of the 4-tuple of functions A, B, F, G was investigated.

Proposition: *The action of $\mathbf{G}$ on the 11-dimensional jet space $P = J^1A \times J^0B \times J^2F \times J^1G$ induces the action of a 9-dimensional Lie group.*

We identified canonical forms in the orbits of the symmetry group for the smoothness classes C^3 and C^ω. Let $a = \frac{\partial A}{\partial x}(\tilde{x})$, $b = B(\tilde{x})$, $f = \frac{\partial^2 F}{\partial x^2}(\tilde{x})$, $g = \frac{\partial G}{\partial x}(\tilde{x})$ be coordinates on the jet space P. Then the canonical forms are described by the following proposition.

Proposition: *Let $q = (a, b, f, g) \in P$. Then there exists an element $\mathbf{g} \in \mathbf{G}$ such that the point $\mathbf{g}(q) = \hat{q} = (\hat{a}, \hat{b}, \hat{f}, \hat{g}) \in P$ satisfies the conditions*

$$\hat{a} = \begin{pmatrix} * & 1 \\ -1 & 0 \end{pmatrix}; \quad \hat{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \hat{f}_{12} = 0, \quad \hat{f}_{11}^2 + \hat{f}_{22}^2 \in \{0, 1\}; \quad \hat{g} = 0. \qquad (3)$$

The value of $\hat{a}_{11}$ depends only on the matrix a and lies in the interval $(-2, 2)$.

Let us denote by Q^* the set of orbits in P which satisfy $\hat{f}_{11}^2 + \hat{f}_{22}^2 = 1$.

Proposition: *Q^* is homeomorphic to the product $\mathbf{R} \times S^1$.*

We found a complete set of invariants, denoted by α, ϕ, which parameterize Q^*. If the values of these parameters pass through certain critical values, then bifurcations of the optimal synthesis occur.

The second chapter deals with the classification of systems of smoothness class C^3. The set Q^* appears to contain three open sets Q_1, Q_2, Q_3, whose union is dense in Q^*. There exist two different topological types of optimal synthesis which correspond to the sets Q_1, Q_2. If (α, ϕ) lies in Q_3 then no optimal synthesis exists.

In **§2.1** we derive necessary and sufficient conditions for the existence of an optimal synthesis for system (1),(2).

Proposition: *Suppose there exists a number $T_* > 0$, a point $x_* \in U$ and a measurable function $u_* : [0, T_*] \to [0, 1]$ such that the trajectories of the system*

$$\dot{x}(t) = A(x(t)) + B(x(t))u_*(t), \qquad x(0) = x_*$$

do not leave the neighbourhood U. Suppose further that $x(T_*) = x(0) = x_*$ and $\int_0^{T_*} (F + u_* G)\, dt < 0$. Then the optimal control problem (1),(2) has no solution.

The inverse is also true.

Proposition: *Suppose that for some point $x^0 \in U$ (and thus for any point in U) there exist a sequence of controls $\{u_n(t)\}$ and numbers $\{T_n\}$ such that the corresponding trajectories $x_n(t)$ with initial value $x_n(0) = x^0$ satisfy*

$$x_n(T_n) = \tilde{x}, \qquad \lim_{n \to \infty} \int_0^{T_n} (F(x_n(t)) + u_n(t) G(x_n(t)))\, dt = -\infty.$$

Then there exists a closed trajectory l, either homeomorphic to S^1 or to a single point, and a corresponding admissible control $u_*(t)$ such that the travel time of the system with control $u_*(t)$ along l is finite and $\oint_l (F + uG)\, dt < 0$.

In **§2.2** we prove a number of auxiliary lemmas, which facilitate further calculations.

In **§2.3** we prove several assertions on the existence and smoothness of invariant submanifolds in the neighbourhood of a degenerated hyperbolic fixed point of a diffeomorphism in the plane.

Theorem: *Let* $M = \{(x, y) \,|\, x \in (0, c],\ y \in [a, b]\} \subset \mathbf{R}^2$ *be a rectangular subset of the plane, where* $c > 0,\ a < b$ *are real numbers. Suppose* $f = (f^x, f^y) : M \to \mathbf{R}_+ \times [a, b]$ *is differentiable and that*

$$\lim_{x \to 0} f^x(x, y) = 0 \quad \forall\, y \in [a, b].$$

Assume further that there exists a constant $L > 0$ *such that*

$$f_x^x \geq 1 + L\|f_y^x\|, \qquad \frac{|f_x^y|}{f_x^x} < L(1 - L\|f_y^x\| - \|f_y^y\|).$$

Here the norm is the C^0*-norm. Upper indices denote the components of* f*, lower indices denote partial derivatives. Then there exists a unique continuously differentiable function* $\gamma : (0, c] \to [a, b]$ *which is Lipschitzian with constant* L*, such that its graph* $\Gamma(\gamma) = \{(x, \gamma(x)) \,|\, x \in (0, c]\} \subset M$ *is invariant under the mapping* f*.*

In **§2.4** the singular trajectories of the system are investigated. The critical values of the parameters α, ϕ, where bifurcations of the optimal synthesis occur, are computed.

In **§2.5** we investigate the smooth Hamiltonian systems in extended phase space which correspond to the extremal control values 0 and 1, in particular their variational equations.

In **§2.6** the optimal synthesis for parameter values in the set Q_1 is described. It contains a one-dimensional singular trajectory.

Theorem: *Let* $(\alpha, \phi) \in Q_1$ *and suppose conditions (3) hold. Without loss of generality suppose* $\tilde{x} = 0$*. Then the optimal synthesis for the optimal control problem (1),(2) exists in the neighbourhood of* $\tilde{x}$ *and has the following structure. In the 3-rd orthant there exists a singular trajectory of order 1, in the 1-st orthant there exists a switching curve from control* $u = 1$ *to control* $u = 0$*. Both curves are transversal to the coordinate axes and join at the point* $\tilde{x}$*. This point is also a singular trajectory of order 1. There exists a unique trajectory* γ_- *that reaches* $\tilde{x}$ *in finite time. Any other trajectory reaches the singular trajectory in finite time and then follow this trajectory. The latter approaches* $\tilde{x}$ *asymptotically in infinite time.*

In **§2.7** the optimal synthesis for parameter values in the set Q_2 is described. It can be characterized as chattering stretched to an infinite time interval. The (nontrivial) critical parameter values on the common boundary of the sets Q_2 and Q_3 are computed.

Theorem: *Let* $(\alpha, \phi) \in Q_2$ *and suppose conditions (3) hold. Without loss of generality suppose* $\tilde{x} = 0$*. Then the optimal synthesis for the optimal control* *problem (1),(2) exists in the neighbourhood of $\tilde{x}$ and has the following structure. In the second orthant there exists a switching curve from $u = 0$ to $u = 1$, in the first orthant a switching curve from $u = 1$ to $u = 0$. Both curves are transversal to the coordinate axes and join at the point $\tilde{x}$. This point is also a singular trajectory of order 1. The trajectories of the system turn around $\tilde{x}$, alternately intersecting the switching curves and changing control. The distance to $\tilde{x}$ and the duration of time intervals with optimal control $u = 1$ decrease exponentially with each turn, approaching asymptotically a geometric progression. The duration of time intervals with optimal control $u = 0$ tends to a positive constant.*

The third chapter deals with the bifurcation of the optimal synthesis corresponding to the common boundary of the sets Q_1 and Q_2. All analytic systems with parameter values α, ϕ on this boundary are classified. It was established that there exists an intermediate type of optimal synthesis, which contains a dispersion curve. Necessary and sufficient conditions on the functions A, B, F, G are found for the realization of the different types of optimal synthesis which occur on this boundary.

In **§3.1** a number of auxiliary lemmas is proven.

In **§3.2** we investigate the region in parameter space which corresponds to an optimal synthesis with a one-dimensional singular trajectory. In this case the optimal synthesis has the following structure.

In the third orthant there exists a singular trajectory γ, which reaches $\tilde{x}$ in finite time. The axis Ox_1 is tangent to γ. In the first orthant there exists a switching curve ρ_{10} from $u = 1$ to $u = 0$. This curve is located transversally to the axes and joins with γ at $\tilde{x}$. Any trajectory that leaves ρ_{10} reaches γ in finite time. The remaining part of the neighbourhood U is filled with trajectories where the control $u = 1$ is optimal. These trajectories reach either ρ_{10} or γ in finite time, except one, which reaches directly the terminal point $\tilde{x}$.

In **§3.3** we investigate the region in parameter space that corresponds to the chattering stretched to an infinite time interval. In this case the optimal synthesis has the following structure.

In the second orthant there exists a switching curve ρ_{01} from $u = 0$ to $u = 1$. In the first orthant there exists a switching curve ρ_{10} from $u = 1$ to $u = 0$. Both curves join at the origin $\tilde{x}$. The curve ρ_{10} is transversal to the axes, while ρ_{01} is tangential to the axis Ox_1. The trajectories of the system turn around $\tilde{x}$, alternately intersecting the switching curves and changing control. The distance to $\tilde{x}$ and the duration of time intervals with optimal control $u = 1$ decrease to zero. The duration of time intervals with optimal control $u = 0$ tends to a positive constant.

In **§3.4** we show the existence of an intermediate type of optimal synthesis, which contains a dispersion curve. In this case the optimal synthesis has the following structure.

In the third orthant there exists a singular trajectory γ. In the second orthant there exists a switching curve ρ_{01} from $u = 0$ to $u = 1$. In the third orthant there exists a dispersion curve ρ_d, which is located between the ray $\{x_1 < 0, x_2 = 0\}$ and the singular trajectory γ. In the first orthant there exists a switching curve ρ_{10} from $u = 1$ to $u = 0$. All these 4 curves join in the point $\tilde{x}$. The first three of them are tangential to the axis Ox_1, while ρ_{10} is transversal to the coordinate axes.

Two families of optimal trajectories, with controls $u = 0$ and $u = 1$ respectively, originate at the dispersion curve ρ_d. The trajectories with control $u = 1$ reach the singular trajectory γ, while the trajectories of the other family first reach the switching curve ρ_{01}, then ρ_{10}, changing the control accordingly, and then also reach γ. Along γ the trajectories reach $\tilde{x}$ in finite time.

In **the fourth chapter** we summarize the results of the previous chapters.

PUBLICATIONS CONTAINING RESULTS OBTAINED IN THIS THESIS

1. Zelikin M.I., Zelikina L.F., Hildebrand R. *Asymptotics of optimal synthesis for one class of extremal problems.* Proceedings of Steklov Mathematical Institute, 233 (2001), pp.87-115

2. Hildebrand R. *Classification of phase portraits of optimal synthesis.* Fundamental and Applied Mathematics, 7 (2001), no.1, pp. 199-233 (in Russian).

3. Hildebrand R. *Optimal synthesis of controlled systems near a focus.* International Congress of Mathematicians 1998, Abstracts of Short Communications and Poster Sessions, Berlin, 1998, p.343

4. Hildebrand R. *Classification of optimal syntheses of controlled systems near a focus.* Workshop on "Mathematical Control Theory and Robotics", Book of Abstracts, SISSA, Trieste, 2000, pp.33-34