Introduction to cryptology (GBIN8U16)

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Password Hashing

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Password hashing as a case-study/illustration

What we have seen so far:

- The importance of (appropriately) modelling security objectives
    - Understanding what we want
    - Making the right assumptions
    - Using the right parameters
- The interest of modular designs

Password hashing has:

- similar needs from “regular” cryptographic hashing
- but in fact quite different!
- $\rightsquigarrow$ pretty different designs in the end when done right (tho may reuse some components)

$\rightsquigarrow$ Let’s have a (rather informal) closer look!

Motivation: How to store a password?

A simple login/password interaction:

1. User U wants to log on system S; sends password p
2. System S checks password associated with U in database $D = \{(U_i, p_i)\}$; grants access if equal to p

A simple total break:

1. Adversary A steals database D (Quite realistic; happens a lot)

⇒ Passwords must never be stored *in clear*!

How to solve this? With Crypto!

A first attempt (aborted):

- Store p encrypted with, say, CTR[E]
- U, S Need to store/know the user-dependent secret key: not much is solved

A first attempt:

- Store p encrypted with a *public* encryption scheme (e.g. RSA-OAEP)
- U needs to know S's public key
- S has a single secret to store (but always used to decrypt; not ideal)

Hash functions to the rescue

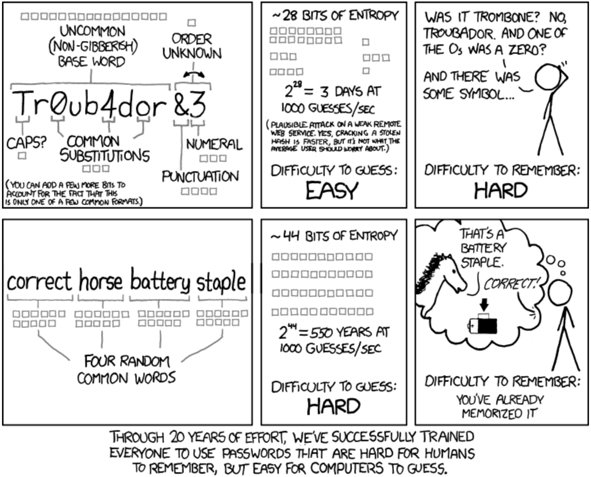
A second atttempt: go keyless!

- Store hashed passwords $\mathcal{H}(p) \rightsquigarrow D = \{(U_i, \mathcal{H}(p_i))\}$
- S checks that the received password hashes to the right value
- If $\mathcal{H}$ is preimage-resistant, $\mathcal{H}(p) \nrightarrow p$?
- Basically sound, but the security analysis is not so simple

Passwords are not random

- Let $\mathcal{H} : \{0,1\}^* \to \{0,1\}^n$. For any explicit set $\mathcal{S}$, $\#\mathcal{S} \lessapprox 2^{n/2}$, $x \in \mathcal{S}$ can be found in time $\leq \#\mathcal{S}$ given $\mathcal{H}(x)$ (Question: why? how?)
- If $\mathcal{H}(x)$ is used to identify x, any preimage works
- “Inverting” $\mathcal{H}$ takes time $\approx \min(2^n, \#\mathcal{S})$ (Assuming $x \twoheadleftarrow \mathcal{S}$)
- Not a problem of hash functions specifically, just the absence of (other) secret

Password entropy: a global issue



`https://xkcd.com/936/`

So, What hash function to use?

Microsoft's LM hash? (1980's)

1. Truncate p to 14 ASCII characters
2. Convert it to uppercase
3. Split it in two halves p_0, p_1
4. $\mathrm{LMHash}(p) = \mathrm{DES}(p_0, c) \| \mathrm{DES}(p_1, c)$ for a fixed constant c
    - $\mathrm{DES} : \{0,1\}^{56} \times \{0,1\}^{64} \to \{0,1\}^{64}$ is a block cipher

What's wrong with that?

- The two halves of the hash are processed separately
- Only $69^7 \lessapprox 2^{43}$ possible inputs per half
    - Only 2^{20} seconds on one core of a typical laptop needed to exhaust them; time-memory tradeoffs are available
- *Impossible* to securely store a strong password

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A better choice: an actual hash function

- A "modern" answer: just take $\mathcal{H}$ to be, say, SHA3-256
- Problem: multi-target attacks are (still) easy
  - An adversary may want to find one password among N
  - For every candidate p', check if $\mathcal{H}(p') \in D$
  - The work is decreased by a factor $\approx N$
  - N might be large (say, > 1000)
- One counter-measure: use different functions for every user
  - Simple to implement: every user U_i selects a large random number r_i (the "salt"); $D = \{(U_i, r_i, \mathcal{H}(r_i \| p_i))\}$ (more generally, may use a "good MAC" with r_i as a key, but not necessary)
  - One has to check for every candidate p', *for every user* if p' is the right password $\rightsquigarrow$ no structural gain from multi-target

But hash functions are too fast!

- If a password is “random enough” (e.g. a 128-bit uniform string), (salted) hashing is fine
- But most/some might not be that
- Assume that one:
  - Has 2^{50} password candidates for a user
  - Can compute 2^{23} hashes/core/second
  - Has 128 available cores
  - $\Rightarrow$ Only 2^{20} seconds ($<$ two weeks) to find p (that’s not enough)
- One counter-measure: make hash functions *slower*
  - Not slow enough to hinder the user
  - Slow enough to make exhaustive search too costly

First slow attempt: PBKDF2

- Instead of computing $\mathcal{H}(r||p)$ once, iterate many times!
- Example: PBKDF2
  - $h \approx \bigoplus_{i=0}^{c} h_i$; $h_i = \mathcal{H}(h_{i-1}||p)$; $h_0 = r$
  - Choose the iteration count c to be “large enough”
  - Typically $c \approx 1000$
- Say it takes 10ms to hash one password ⇒ 35 years on 10 000 cores to try 2^{50} candidates for one user
- One problem:
  - The user *needs* to hash on a regular core
  - An adversary may try hashes on fast dedicated circuits

Selective slowness

A reasonable assumption:

- A PBKDF2 hash function can be computed 2^{20} times faster than on a CPU core by using dedicated hardware with low amortized cost
- 10ms to hash one password on CPU $\Rightarrow < 2^{-26}$s on efficient hardware $\Rightarrow < 2^{20}$ seconds on 10 machines to try 2^{50} passwords

How to solve this?

- Cannot make the user wait one day to check a password
- So use hashing that's *slow everywhere*

What’s slow anyway?

An assumption: memory is similarly slow for everybody (CPU, GPU, FPGA, ASIC)

- So use a “memory-hard” hash function that needs a lot of memory to be computed
- A framework: the output must depend on “many” intermediate values, accessed many times $\rightsquigarrow$ a (quadratic) tradeoff
  - Either store all intermediate values (costs memory)
  - Or recompute them as needed (costs time)
- Only increases memory consumption (not time) of hashing a password for a generic user
- Makes dedicated hardware not more efficient than regular CPU (hopefully)

One memory-hard example: scrypt

Scrypt (Percival, 2009), the (very rough) idea:

- Use the password and salt to generate a large buffer
- Access the buffer many times in an unpredictable way to generate the output

A bit more precisely:

1. $h_i = \mathcal{H}(h_{i-1})$; $h_0 = r||p$, for i up to $n-1$
2. $s_i = \mathcal{H}(s_{i-1} \oplus h_{s_{i-1} \bmod n})$, $s_0 = \mathcal{H}(h_{n-1})$, for i up to n
3. Return s_n

Scrypt comments

The intuitive tradeoff from two slides ago becomes:

- Either store all the h_i's $\rightsquigarrow$ time $=$ memory $\approx n$ calls to $\mathcal{H}/$accesses
- Either recompute $h_{s_{i-1} \bmod n}$ once s_{i-1} is known $\rightsquigarrow$ constant memory, time $\approx n \times n/2$ calls to $\mathcal{H}$
- Any combination in between (e.g. store one tenth of the h_i's, regularly spaced)

⇒ Only a few MB of generated values might be enough to defeat special-purpose hardware

- One can in fact prove that the above tradeoff is roughly optimal (Alwen & al., 2016)

An alternative approach: “Halting puzzles”

HKDF (Boyen, 2007) uses a memory-hard function with an (optionally) *unknown* iteration count

1. A user computes an iterated function on the password p
2. Interrupts the process when wanted; obtains a hash h of p and a verification string v
3. The hash and the iteration count can be retrieved from p and v

- The user may tune the iteration count on its own to its requirements
- Without that knowledge, an adversary is less efficient

HKDF: How?

Preparation phase:
Input: p, r, t
Output: h, v, r

```
$z = \mathcal{H}(r\|p)$
For $i = 1, \ldots, t$ $\triangleleft$ $t$ may be user-defined
    $y_i = z$
    For $* = 1, \ldots, q$ $\triangleleft$ $q$ controls the time/space ratio
        $j = 1 + (z \mod i)$
        $z = \mathcal{H}(z\|y_j)$
Return $r$; $v = \mathcal{H}(y_1\|z)$; $h = \mathcal{H}(z\|r)$
```

HKDF: How? (bis)

Extraction phase:
Input: p, r, v
Output: h

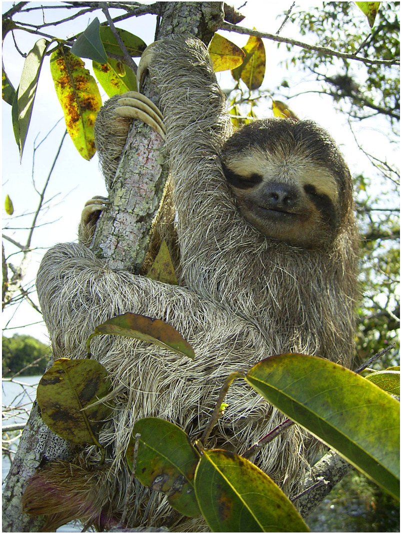
```
$z = \mathcal{H}(r \| p)$
For $i = 1, \ldots, \infty$
    $y_i = z$
    For $* = 1, \ldots, q$
        $j = 1 + (z \mod i)$
        $z = \mathcal{H}(z \| y_j)$
        If $(\mathcal{H}(y_1 \| z) = v)$ Then Break
Return $h = \mathcal{H}(z \| r)$
```

HKDF, Scrypt comments

- Both functions use password-dependent memory accesses
- May leak information about the password (via side-channels)
- So (memory-hard) functions with password-independent accesses may sometimes be preferable
  - But then an adversary could set up good “dedicated” tradeoffs $\rightsquigarrow$ careful in picking the access pattern

- For more on password hashing: `https://password-hashing.net/`

To finish: something a bit different



To finish: something a bit different

It may be useful to have a hash function that:

- Is slow to execute (i.e. it is slow to compute $y := \mathcal{H}(x)$ given x)
- Is fast to verify (i.e. it is fast to check that $y = \mathcal{H}(x)$ given x and y)
- $\rightsquigarrow$ *Verifiable delay functions* (VDF)

An application:

- Collaborative random-number generation

Public randomness

Randomness beacon

A *Randomness beacon* is a system that publishes (pseudo-)random numbers at regular interval

Example:

- `https://beacon.nist.gov/home`

Some applications:

- Remote random consensus ("Shall we go to a pizzeria or a crêperie?")
- (Faster) challenge generation in authentication protocols
- Lotteries
- Jury/assembly selection
- Non-deterministic voting schemes

Collaborative beacons

One can distinguish:

- "Oracle" beacons (have to be trusted)
- "Collaborative" beacons (everyone can contribute)

A design strategy (Lenstra & Wesolowski, 2015):

1. Use a slow hash function with fast verification that takes wall time $> \Delta$ to be computed (hopefully on the best platform)
2. Gather public seeds from time $t - \Delta$ to t
3. At time t, hash all collected seeds, then publish the hash
4. Everyone can efficiently test the result and its dependence on the seeds

- An adversary does not have time to precompute a hash and insert a seed that biases the result

A candidate slow hash function

Sloth: A slow hash function in a nutshell:

- If $p \equiv 3 \mod 4$ is a (large) prime, if $x \in \mathbb{F}_p^\times$ is a square mod p, the fastest know way to compute a square root of x is as $x^{(p+1)/4}$
- Exactly one of x or $-x$ is a square (knowing which is easy) $\Rightarrow$ one can map any number to a well-defined square root
- Computing a square root takes $\approx \log(p)$ more time than "verifying" one

So (to make things more modular):

- Compute an iterative chain of square roots
- Interleaved with, say, block cipher applications to break the algebraic structure

Some comments

- Sloth is not memory-hard, but CPUs are good at big-number arithmetic
  - Dedicated hardware may not be a threat
  - (Some password-hashing functions are based on the same assumption (Pornin, 2014))
- A Twitter-accessible beacon (not really tweeting anymore): `https://twitter.com/random_zoo`
- The computation/verification gap in Sloth is not great asymptotically; better functions exist (cf. e.g. Wesolowski, 2019)