Crypto Engineering '20

Hash functions

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Hash functions as a figure

$\rightsquigarrow$ on the board

First definition

Hash function

A hash function is a mapping $\mathcal{H} : \mathcal{M} \to \mathcal{D}$

So it really is just a function...

Usually:

- $\mathcal{M} = \bigcup_{\ell < N} \{0,1\}^\ell$, $\mathcal{D} = \{0,1\}^n$, $N \gg n$
- N is typically $\geq 2^{64}$, $n \in \{$~~128~~, ~~160~~, ~~224~~, 256, 384, 512$\}$

Also popular now: extendable-output functions (XOFs): $\mathcal{D} = \bigcup_{\ell < N'} \{0,1\}^\ell$

- Hash functions are *keyless*
- So, how do you tell if one's **good**?

Idealized hash functions: Random oracles

Random oracle

A function $\mathcal{H} : \mathcal{M} \to \mathcal{D}$ s.t. $\forall x \in \mathcal{M}$, $\mathcal{H}(x) \xleftarrow{\$} \mathcal{D}$

- "The best we can ever get"
- Sometimes useful in proofs ("Random oracle model", or ROM)
- Not possible to have one (except for small (co-)domains assuming a TRNG)
- But we can get *approximations* (e.g. SHA-3)
- Equivalent to the Ideal Cipher Model (Coron et al., 2008; + later patches)

Main security properties

What is hard for a RO should be hard for any HF

$\Rightarrow$

1. **First preimage**: given t, find m s.t. $\mathcal{H}(m) = t$
2. **Second preimage**: given m, find $m' \neq m$ s.t. $\mathcal{H}(m) = \mathcal{H}(m')$
3. **Collision**: find $(m, m' \neq m)$ s.t. $\mathcal{H}(m) = \mathcal{H}(m')$

Generic complexity:
1), 2): $\Theta(2^n)$;
3): $\Theta(2^{n/2})$ $\leftsquigarrow$ “Birthday paradox”

Why do we care? Applications!

Hash functions are useful for:

- Hash-and-sign (RSA signatures, (EC)DSA, ...)
- Message-authentication codes (HMAC, ...)
- Password hashing (with a grain of salt)
- Hash-based signatures (inefficient but PQ)
- As "RO instantiations" (OAEP, ...)
- As one-way functions (OWF)

So, how do you build hash functions?

- Objective #1: be secure
- Objective #2: be efficient
  - At most a few dozen cycles/byte!
  - $\Rightarrow$ work with limited amount of memory

So...

- (#2) Build $\mathcal{H}$ from a *small component*
- (#1) Prove that this is okay

What kind of small component?

Compression function

A compression function is a mapping $f : \{0,1\}^n \times \{0,1\}^b \to \{0,1\}^n$

- A family of functions from n to n bits
- Not unlike a block cipher, only not invertible

Permutation

A permutation is an invertible mapping $p : \{0,1\}^n \to \{0,1\}^n$

Yes, very simple

- Like a block cipher with a fixed key, *e.g.* $p = \mathcal{E}(0, \cdot)$

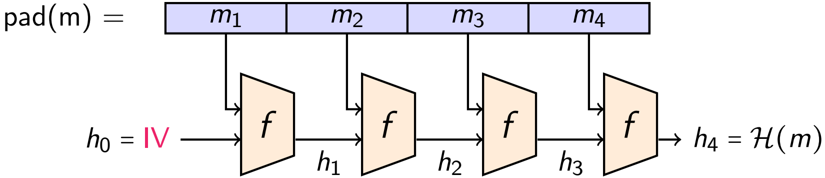
From small to big (compression function case)

Assume a good f

- Main problem: fixed-size domain $\{0,1\}^n \times \{0,1\}^b$
- Objective: *domain extension* to $\bigcup_{\ell < N}\{0,1\}^\ell$
- (Not unlike using a mode of operation with a BC)

The classical answer: the Merkle-Damgård construction (1989)

MD: with a picture



That is: $\mathcal{H}(m_1 \| m_2 \| m_3 \| \ldots) = f(\ldots f(f(f(\mathrm{IV}, m_1), m_2), m_3), \ldots)$

$\mathrm{pad}(m) \approx m \| 1000\ldots00\langle \text{length of } m \rangle \leftarrow$ *stengthening*

MD: does it work?

Efficiency?

- Only sequential calls to f
- $\Rightarrow$ fine

Security?

- Still to be shown
- Objective: *reduce* security of $\mathcal{H}$ to that of f
  - "If f is good, then $\mathcal{H}$ is good"
- True for collision and first preimage, **false** for second preimage

MD (partial) security proof

Method: simple contrapositive argument

- Attack $\{1^{st}$preim., coll.$\}$ on $\mathcal{H} \Rightarrow$ attack $\{1^{st}$preim., coll.$\}$ on f

First preimage case

If $\mathcal{H}(m_1 \| m_2 \| \ldots \| m_\ell) = t$, then $f(\mathcal{H}(m_1 \| m_2 \| \ldots \| m_{\ell-1}), m_\ell) = t$

Collision case (sketch)

If $\mathcal{H}(m_1 \| m_2 \| \ldots \| m_\ell) = \mathcal{H}(m'_1 \| m'_2 \| \ldots \| m'_\ell)$, show that $\exists i$ s.t. $(h_i := \mathcal{H}(m_1 \| m_2 \| \ldots \| m_{i-1}), m_i) \neq (h'_i := \mathcal{H}(m'_1 \| m'_2 \| \ldots \| m'_{i-1}), m'_i)$ and $f(h_i, m_i) = f(h'_i, m'_i)$

- Proper message padding (such as stenghtening) necessary to make it work!

What about 2nd preimages??

No proof (with optimal resistance), can't have one:

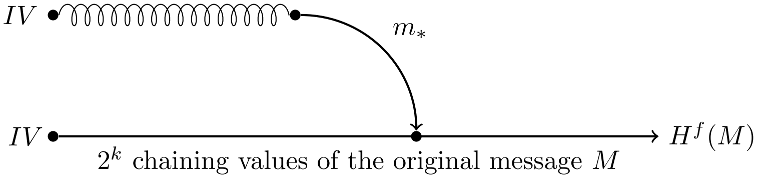
- Generic attack on messages of 2^k blocks for a cost $\approx k2^{n/2+1} + 2^{n-k+1}$ (Kelsey and Schneier, 2005)
- Idea: exploit internal collisions in the h_is

This is not nice, but:

- Requires (very) long messages to gain something
- At least as expensive as collision search
  - Always going to be the case, as preimage $\Rightarrow$ collision
- If n is chosen s.t. generic collisions are out of reach, we're somewhat fine

$\Rightarrow$ Didn't make people give up MD hash functions (MD5, SHA-1, SHA-2 family)

Attack with an expandable message



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Is that unavoidable?

No! Simple patch: Chop-MD/Wide-pipe MD (Coron et al., 2005) and (Lucks, 2005)

- Build $\mathcal{H}$ from $f : \{0,1\}^{2n} \times \{0,1\}^b \to \{0,1\}^{2n}$, truncate output to n bits (say)
- Collision in the output $\nRightarrow$ collision in the internal state
- Very strong provable guarantees (Coron et al.)
  - Secure domain extender for fixed-size RO
- Concrete instantiations: SHA-512/224, SHA-512/256 (2015)

But careful with models!

- Coron et al. show very strong provable guarantees for Chop-MD
  - Secure domain extender for fixed-size RO
- But this in fact doesn't guarantee weaker ones, such as preservation of collision-resistance (Bellare & Ristenpart, 2006)!
  - One can do "stupid things" with a non-ideal compression function
  - $\rightsquigarrow$ Chop-MD with a (real) CR c.f. is not (necessarily) CR!
  - (In essence, one needs strengthening in the padding)

Practical impact of the MD proof

- If one can't attack f underlying $\mathcal{H}$, all is well
- Else, ...???
- $\Rightarrow$ Attacking f is a meaningful goal for cryptographers ($\approx$ (semi-)freestart attacks)
- Ideally, *never* use a $\mathcal{H}$ with broken f

The MD5 failure

- MD5: designed by Rivest (1992)
- 1993: very efficient collision attack on the compression function (den Boer and Bosselaers); mean time of 4 minutes on a 33 MHz 80386
- MD5 still massively used...
- 2005: very efficient collision attack on the *hash function* (Wang and Yu)
- Still used...
- 2007: practically threatening collisions (Stevens et al.)
- Still used...
- 2009: even worse practical collision attacks (Stevens et al.)
- Hmm, maybe we should move on?

Was this avoidable?

Yes!

- Early signs of weaknesses ⇒ move to alernatives ASAP!
- What were they (among others)?
  - 1992: RIPEMD (RIPE); practically broken (collisions) 2005 (Wang et al.)
  - 1993: SHA-0 (NSA); broken (collisions) 1998 (Chabaud and Joux); practically broken 2005 (Biham et al.)
  - 1995: SHA-1 (NSA); broken (collisions) 2005 (Wang et al.); practically broken 2017 (Stevens et al. (and me!))
  - 1996: RIPEMD-128 (Dobbertin et al.); broken (collisions) 2013 (Landelle and Peyrin)
  - 1996: RIPEMD-160 (Dobbertin et al.); unbroken so far
  - 2001: SHA-2 (NSA); unbroken so far

Lesson to learn?

- Don't use broken algorithms
- Care about "theoretical" attacks

Perfect bad example: Git

- Don't use SHA-1 in 2005!
- Don't hide needed security properties!

Also:

- Don't use SHA-1, even if you just care about preimage attacks

Back to design: how to do f?

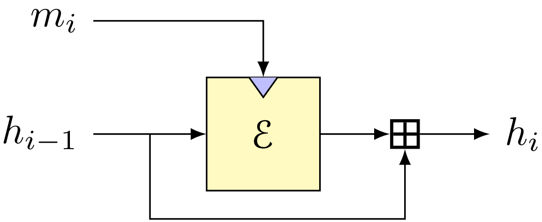
1. Start like a block cipher
2. Add feedforward to prevent invertibility

Examples:
"Davies-Meyer": $f(h, m) = \mathcal{E}_m(h) \boxplus h$
"Matyas-Meyer-Oseas": $f(h, m) = \mathcal{E}_h(m) \boxplus m$

- Systematic analysis by Preneel, Govaerts and Vandewalle (1993). "PGV" constructions
- Then rigorous proofs (in the ideal cipher model) (Black et al., 2002), (Black et al., 2010)

Re: Davies-Meyer

Picture:



Used in MD4/5 SHA-0/1/2, etc.

Re: Re: Davies-Meyer

Why is the "message" the "key"?

- We're not encrypting with $\mathcal{E}$ so we can do what we want
- Disconnect chaining value and message length!
- $\mathcal{E}$'s block length: fixed by security level
- $\mathcal{E}$'s key length: fixed by "message" length
- Large "key" $\Rightarrow$ more efficient
- Example: MD5's "block cipher" (also bad): 128-bit blocks, 512-bit keys

DM incentive: use very simple *message expansion* ("key schedules")

- To be efficient!
- Warning: can be a source of weakness (MD5, SHA-0, SHA-1. Should that be surprising?)

Major PGV Warning

PGV constructions are proved secure in the *ideal cipher model*, **BUT**

- Real ciphers are not ideal!
- Real ciphers *don't have to be ideal* to be okay ciphers (e.g. a "good" PRP is usually enough)
  - IDEA (Lai and Massey, 1991): weak key classes (Daemen et al., 1993)
  - TEA (Wheeler and Needham, 1994): equivalent keys (Kelsey et al., 1996)

What can go wrong?

Bad case of crypto design

Microsoft needed a hash function for ROM integrity check of the XBOX

- Used TEA in DM mode (Steil, 2005)
- Because of an earlier break of their RC4-CBC-MAC scheme (ibid.)
- Terrible idea, because of existence of equivalent keys!
- $\text{TEA}(k, m) = \text{TEA}(\hat{k}, m) \Rightarrow \text{DM-TEA}(h, k) = \text{DM-TEA}(h, \hat{k}) \Rightarrow$ easy collisions!
- Got hacked...
- IDEA for a hash function: also bad (Wei et al., 2012)

NEVER DESIGN YOUR OWN CRYPTO!

It’s not all that bad, tho

- AES in a PGV construction so far unbroken (see *e.g.* Sasaki (2011))
  - But small parameters ‽
- Ditto, SHA-256 as a block cipher: “SHACAL-2” (Handschuh and Naccache, 2001)
  - Enormous keys, 512 bit!

Now just a few examples

The MD4/MD5/SHA-0/SHA-1 family

- Merkle-Damgård construction
- Davies-Meyer compression function
  - 512-bit messages, 128-bit (resp. 160-bit) chaining values for MD4/5 (resp. SHA-0/1)
  - Very simple message expansion
- All broken (MD4 could be broken *by hand*)

RIPEMD family

- Somewhat similar, but uses two parallel lines merged at the end

SHA-2

- Somewhat similar, but heavier message expansion

And now for something different

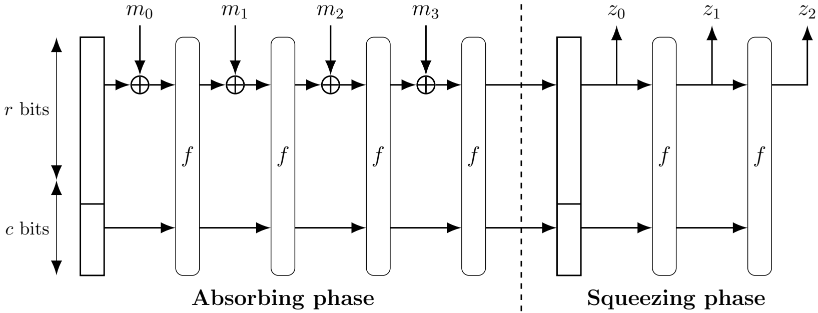
If you need a hash function today ⇒ SHA-3 (initially Keccak, (Bertoni et al., 2008))

- Winner of an academic competition run by NIST (2008–2012)
- Sponge construction (not Merkle-Damgård)
- Based on a permutation (not a compression function)

Sponge:

1. Compute $i := p(p(\ldots p(m_1 \| 0^c) \oplus m_2 \| 0^c) \ldots)$
2. Output $\mathcal{H}(m) := \lfloor i \rfloor_r \| \lfloor p(i) \rfloor_r \| \ldots \| \lfloor p^n(i) \rfloor_r$

Picture of a sponge



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Sponge nice features

- Indifferentiable from a RO (same, as Wide-pipe MD) (Bertoni et al., 2008)
- Quite flexible
  - For fixed permutation size: speed/security tradeoff
- Natively a XOF
- Can be extended to do (authenticated) encryption
- Simpler to design a permutation; less of a waste?

- Close structure: JH construction, another SHA-3 competitor (Wu, 2008)

Conclusion

- Don't design crypto yourself!
  - There is no generic way to design a hash function
  - Every small detail counts (recall e.g. SHA-0, TEA)
- Use SHA-3 (SHA-2 still okay)
- NEVER USE MD5/SHA-1
  - Even if you only care about preimage attacks