

New instantiations of the CRYPTO 2017 masking schemes

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Masking schemes for finite field multiplication

Proving security

New instantiations of the schemes from CRYPTO 2017

Conclusion

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The context

Context: Crypto implementation on observable devices

Objective: **secure finite-field multiplication w/ leakage**

- ▶ Implement $(a, b) \mapsto c = a \times b$, $a, b, c \in \mathbb{K}$
 - ▶ Used in non-linear ops in sym. crypto (e.g. S-boxes)
 - ▶ Input/outputs usually secret!
- ▶ Problem: **computations leak information**
- ▶ \leadsto Need a way to compute a product w/o leaking (too much) the operands & the result
- ▶ Our focus: **higher-order** (many shares) **software schemes** (no glitches)

Basic idea

- ▶ Split a, b, c into *shares* (i.e. use a secret-sharing scheme)
 - ▶ Typically simple and additive:
$$x = \sum_{i=0}^d x_i, x_0, \dots, x_{d-1} \stackrel{s}{\leftarrow} \mathbb{K}, x_d = x - \sum_{i=0}^{d-1} x_i$$
- ▶ Compute the operation over the shared operands; obtain a shared result
- ▶ Ensure that neither of a, b, c can be (easily) recovered

Prove security e.g. in:

- ▶ **The probing model** \rightsquigarrow d -privacy (Ishai, Sahai & Wagner, 2003) / d -(S)NI (Belaïd et al., 2016)
- ▶ The noisy leakage model (Chari et al. '99, Prouff & Rivain, 2013)
- ▶ (For relations between the two, see e.g. Dahmun's talk this afternoon)

First attempt

- ▶ We want to compute $c = \sum_k c_k = \sum_i a_i \times \sum_j b_j = \sum_{i,j} a_i b_j$
- ▶ So maybe define $c_i = a_i \sum_{j=0}^d b_j$?
- ▶ Problem: any single c_i reveals information about b
- ▶ One solution (ISW, 2003): **rerandomize** using fresh randomness
 - ▶ For instance (for $d = 3$):
 - ▶ $c_0 = a_0 b_0 + r_{0,1} + r_{0,2} + r_{0,3}$
 - ▶ $c_1 = a_1 b_1 + (r_{0,1} + a_0 b_1 + a_1 b_0) + r_{1,2} + r_{1,3}$
 - ▶ $c_2 = a_2 b_2 + (r_{0,2} + a_0 b_2 + a_2 b_0) + (r_{1,2} + a_1 b_2 + a_2 b_1) + r_{2,3}$
 - ▶ $c_3 =$
 $a_3 b_3 + (r_{0,3} + a_0 b_3 + a_3 b_0) + (r_{1,3} + a_1 b_3 + a_3 b_1) + (r_{2,3} + a_2 b_3 + a_3 b_2)$
- ▶ Prove security in the probing model
- ▶ † **Scheduling of the operations is important** (impacts the probes available to the adversary), hence the (\cdot) s

Masking complexity

- ▶ ISW provides a practical solution for masking a multiplication
- ▶ But the cost is **quadratic in d** : d -privacy requires:
 - ▶ $2d(d+1)$ sums
 - ▶ $(d+1)^2$ products
 - ▶ $d(d+1)/2$ fresh random masks
- ▶ Decreasing the cost/overhead of masking is a major problem
 - ▶ Use block ciphers that need few multiplications (e.g. ZORRO, Gérard et al., 2013 (broken))
 - ▶ Amortize the cost of masking several mult. (e.g. Coron et al., 2016)
 - ▶ Decrease the cost of masking a single mult. (e.g. Belaïd et al., 2016, 2017)

Two schemes introduced by Belaïd et al. (2017):

- ▶ “Alg. 4”, with **linear bilinear multiplication complexity**, requiring:
 - ▶ $9d^2 + d$ sums
 - ▶ $2d^2$ linear products
 - ▶ $2d + 1$ products
 - ▶ $2d^2 + d(d - 1)/2$ fresh random masks
- ▶ “Alg. 5”, with **linear randomness complexity**, requiring:
 - ▶ $2d(d + 1)$ sums
 - ▶ $d(d + 1)$ linear products
 - ▶ $(d + 1)^2$ products
 - ▶ d fresh random masks

Focus on Alg. 4

This scheme uses shares of three kinds:

- ▶ $c_0 := (a_0 + \sum_{i=1}^d (r_i + a_i)) \cdot (b_0 + \sum_{i=1}^d (s_i + b_i));$
- ▶ $c_i := -r_i \cdot (b_0 + \sum_{j=1}^d (\delta_{i,j} s_j + b_j)), 1 \leq i \leq d;$
- ▶ $c_{i+d} := -s_i \cdot (a_0 + \sum_{j=1}^d (\gamma_{i,j} r_j + a_j)), 1 \leq i \leq d.$

With:

- ▶ $\gamma = (\gamma_{i,j}) \in \mathbb{K}^{d \times d}$
- ▶ $\delta = (\delta_{i,j}) \in \mathbb{K}^{d \times d}$ s.t. $\gamma + \delta$ is the all-one matrix

(Plus an additional post-processing, not studied here)

Problem: finding γ so that the scheme is *secure* is hard. Belaïd et al.:

- ▶ Found an explicit γ for $d = 2$ over \mathbb{F}_{2^2} (and other larger fields)
- ▶ Proved (non-constructively) the existence of good γ at order d over \mathbb{F}_q when $q > \mathcal{O}(d)^{d+1}$

Our results: we give constructions/examples for:

- ▶ $d = 3$ over \mathbb{F}_{2^k} , $k \geq 3$
- ▶ $d = 4$ over \mathbb{F}_{2^k} , $5 \leq k \leq 16$
- ▶ $d = 5$ over \mathbb{F}_{2^k} , $10 \leq k \leq 16$
- ▶ $d = 6$ over \mathbb{F}_{2^k} , $15 \leq k \leq 16$

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What's a good γ anyways?

To attack Alg. 4, one typically wants to:

- 1 Select d probes p_0, \dots, p_{d-1} of intermediate values
- 2 Find \mathcal{F} s.t. the distribution of $\mathcal{F}(p_0, \dots, p_{d-1})$ depends on a (say)

In Alg. 4, the possible probes (relating to a) are:

- ▶ $a_i, r_i, a_i + r_i, \gamma_{j,i}r_i, a_i + \gamma_{j,i}r_i$, for $0 \leq i \leq d, 1 \leq j \leq d$
- ▶ $a_0 + \sum_{i=1}^k (a_i + r_i)$, $1 \leq k \leq d$
- ▶ $a_0 + \sum_{i=1}^k (a_i + \gamma_{j,i}r_i)$, $1 \leq k \leq d, 1 \leq j \leq d$

Proposition: it is sufficient to only consider \mathcal{F} s that are **linear combinations of the p_i s** (cf. Belaïd et al., 2017)

Attack sets

One sub-objective: decide if a set of probes P leads to an attack

- ▶ For each probe, consider indicator vectors of \mathbf{l} of its a_i s and \mathbf{m} of its r_i s
- ▶ E.g. $a_0 + a_1 + \gamma_{1,1}r_1$ ($d = 2$) \rightsquigarrow

$$\mathbf{l} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 0 \\ \gamma_{1,1} \\ 0 \end{pmatrix}$$

- ▶ Gather all such vectors in larger matrices \mathbf{L}_P and \mathbf{M}_P^γ
- ▶ Attack: find x_i s s.t. $\pi := \sum x_i p_i = \sum y_i a_i + \sum z_i r_i$ with $y_i \neq 0$, $z_i = 0$ for all i
 - ▶ If π “includes an r_i ” or “misses an a_i ”, then it is uniform
- ▶ So there is an attack iff. $\exists u \in \ker \mathbf{M}_P^\gamma$ s.t. $\mathbf{L}_P u$ is of full weight

Immediate algorithm

To prove security for a given γ :

- ▶ Look at all matrices \mathbf{L}_P and \mathbf{M}_P^γ for d probes P
- ▶ For each:
 - 1 Compute a basis \mathbf{B} of the (right) kernel of \mathbf{M}_P^γ
 - 2 There is an attack with P iff. $\mathbf{N}_P := \mathbf{L}_P \mathbf{B}$ has no all-zero row
- ⇐ If \mathbf{N}_P has a zero row, then no linear combination of probes depends on all a_i s and cancels all r_i s
- ⇒ If \mathbf{N}_P has no zero row, there is at least one linear combination of probes that depends on all a_i s and cancels all r_i s
 - ▶ By a combinatorial argument, as long as $\#\mathbb{K} > d$ (e.g. use Schwartz-Zippel-DeMillo-Lipton)

Testing optimizations

The previous algorithm allows to test the security of an instance by checking $\approx \binom{d^2}{d}$ (!) matrices $\mathbf{L}_P, \mathbf{M}_P^\gamma$. Some optims:

- ▶ Do early-abort
- ▶ Check “critical cases” first
- ▶ Don't check stupid choices for P
- ▶ Use batch kernel computations

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Finding secure instantiations

The testing algorithm can be used to find secure instantiations:

- 1 Draw γ (δ) at random
- 2 Check that there is no attack

It works, but we can do better by picking super-regular/MDS γ s (δ s) \leftarrow All square submatrices invertible

Observations:

- ▶ If $\dim \ker \mathbf{M}_P^\gamma = 0$, then no attack is possible w/ probes P
 - ▶ Try to pick γ s.t. \mathbf{M}_P^γ is invertible for many P s
- ▶ Many \mathbf{M}_P^γ 's are made of submatrices of γ
 - ▶ All invertible, if γ is MDS
- ▶ (Additionally: ensure invertibility w/ added columns of 1 \rightarrow “XMDS” matrices)

MDS precondition: small cases

- ▶ For $d = 1, 2$, it is sufficient for γ, δ to be XMDS for the scheme to be secure
- ▶ For $d = 3$, one must **additionally check** that no matrix of the form

$$\begin{pmatrix} \gamma_{i,1} & \gamma_{j,1} & \gamma_{k,1} \\ \gamma_{i,2} & \gamma_{j,2} & \gamma_{k,2} \\ \gamma_{i,3} & \gamma_{j,3} & 0 \end{pmatrix}, i \neq j \neq k,$$

is singular

- ▶ Not systematically ensured by the XMDS property
- ▶ Can be **solved symbolically**

XMDS precondition: larger cases; enforcement

- ▶ For $d \geq 4$, not feasible (?) to enforce invertibility of all \mathbf{M}_P^γ
- ▶ But XMDS γ s are still more likely to be secure than non-XMDS ones
 - ▶ E.g. w/ Pr 0.063 instead of 0.030 for $d = 4$ over \mathbb{F}_{2^8}
- ▶ Problem: how to ensure that *both* γ and δ are XMDS?
 - ▶ Use a (generalized) Cauchy construction $x_{i,j} = c_i d_j / (x_i - y_j)$, viz. $\gamma_{i,j} = x_i / (x_i - y_j)$
 - ▶ Then $\delta_{i,j} = 1 - x_i / (x_i - y_j) = -y_j / (x_i - y_j)$, so δ is Cauchy and then (X)MDS

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The end?

- ▶ We found more instances of the (two) masking schemes of CRYPTO 2017, at larger orders
- ▶ Still only reaching $d = 4$ over “useful” fields such as \mathbb{F}_{2^8}
- ▶ \Rightarrow Still room for improvements