Assimilation of Remote Sensed Data for River Hydraulic Simulations

Jérôme Monnier[1,*], **Marc Honnorat**[1], **Xijun Lai**[2], and **Francois-Xavier Le Dimet**[1]

[1] Univ. of Grenoble (INPG-INRIA-UJF), LJK - Moise project-team, France.
[2] Nandging Institute Geography Limnulogy, Chineese Ac. Sc., 210008, China.

We present variational data assimilation methods applied to river hydraulics, especially when flooding. We consider two kinds of configurations. First, if observations are lagrangian trajectories (either extracted from videos or GPS drifting buoys); second, if observations include one satellite image of flood plain. Numerical results (with synthetic or real data) are presented.

1 Introduction

In river hydraulics, observations are available only in very small quantities. Classically, water levels are measured at gauging stations but they are very sparse in space and only inside the main channel. Velocity measurements are even more scarce and uncertain; they usually require complex human interventions. Consequently, in order to take full advantages of data assimilation for river and flooding models, we think to use remote sensing data which can bring extra information on the flow. Here, we present a synthesis of two studies related to variationnal data assimilation methods for river flows when remote sensed data are available. In section 2, we present the mathematical models (St-Venant 1.5D and 2D), the cost function to be minimized and the full 4D-var method implemented into our software DassFlow. In Section 3, we present a real flood event (Moselle river) where we identify inflow discharge when one satellite image and pointwise elevation values are partially available. In Section 4, we present the assimilation of surface flow trajectories (lagrangian data).

2 Mathematical models

The 1.5D forward model relies on the St-Venant equations (shallow-water assumption) with extra lateral fluxes (modelling over-flowing):

$$\begin{cases} \dfrac{\partial S}{\partial t} + \dfrac{\partial Q}{\partial \tilde{x}} = q_{\mathbf{n}} \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial \tilde{x}}\left(\dfrac{Q^2}{S} + P\right) - g\dfrac{\partial b}{\partial \tilde{x}}\dfrac{H^2}{2} + gS\dfrac{\partial z_b}{\partial \tilde{x}} = q_{\mathbf{n}}\dfrac{Q}{S} + s_f \end{cases} \tag{1}$$

where S is the wetted area, Q the lineic-discharge, b cross-section width and s_f the 1D friction term. The extra source terms q_n model potential over-flowing. When coupling, q_n is given by the following 2D model. The system is closed with initial conditions and boundary conditions. The 2D forward model relies on the 2D St-Venant equations:

$$\begin{cases} \partial_t h + \mathrm{div}(\mathbf{q}) = 0 \\ \partial_t \mathbf{q} + \mathrm{div}\left(\frac{1}{h}\mathbf{q}\otimes\mathbf{q}\right) + \frac{1}{2}g\nabla h^2 + gh\nabla z_b + g\frac{n^2\|\mathbf{q}\|_2}{h^{7/3}}\mathbf{q} = 0 \\ h(0) = h_0, \qquad \mathbf{q}(0) = \mathbf{q}_0 \end{cases} \tag{2}$$

where h is the elevation and q the discharge, z_b the bed elevation, n the Manning roughness coefficient. Boundary conditions are: at inflow, the discharge is prescribed; at open boundaries or outflow, incoming characteristics are prescribed; walls conditions. Given the control vector k=(parameters, initial conditions, boundary conditions), the state variables are determined by solving the forward models. Then, we can compute the cost function:

$$J(\mathbf{k}) = \alpha_{obs}J_{obs}(\mathbf{k}) + \alpha_{flux}J_{flux}(\mathbf{k}) + \alpha_{reg}J_{reg}(\mathbf{k}) \tag{3}$$

where J_{obs} measures the discrepancy between data and simulated state, J_{reg} is a regularisation term and J_{flux} is an unsual pseudo-observation term which measures the flux discrepancy when elevation measures only are available. The optimization problem consists to minimize $J(k)$ with respect to k. To this end, we use a local descent algorithm (quasi-Newton method, BFGS) . The gradient of the cost function is computed using the adjoint method. All the process is implemented into our software DassFlow [2].

* Corresponding author E-mail: Jerome.Monnier@imag.fr, Phone: +00 33 476 635963, Fax: +00 33 476 631263

3 One satellite image of flood plain

We consider a real flood event (of 66h) at Moselle river, feb. 1997. Water levels are available in the main channel (at EDF station), see Fig. 1, and one satellite image is available too. After post-processing of the image [6], one obtains classical water elevation values distributed in space and pointwise in time, Fig.1. Using the 4D-var method, the extra satellite image allows to calibrate much better than by hand the Manning roughness coefficients n (in 10 different land-uses). Also, twin experiments show that if water elevations available at EDF stations were more dense in time and the image more dense in space, we could have retreived the incoming inflow discharge of the event (ie the flood wave).

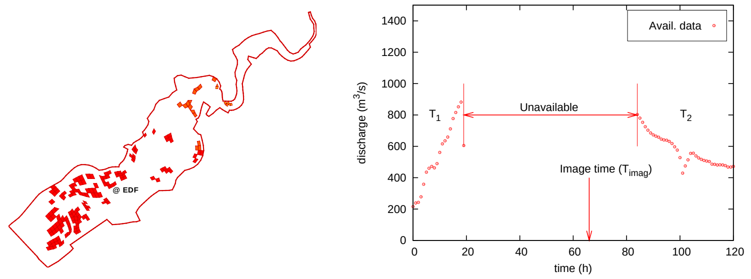

Fig. 1 a) h values extracted from satellite image. b). h values available at EDF station. Courtesy of Cemagref (C. Puech, R. Hostache).

4 Lagrangian data

In some configurations, one can expect lagrangian trajectories as observations; for example either from GPS drifting floaters, see figure (large scale), or extracted from video images over a canal (small scale). In both cases, after filtering (small scale turbulence, secondary currents etc, depending on the flow), the trajectories of particles advected by the flow can be used in addition to classical eulerian observations. This lagrangian data brings extra information on the velocity flow thanks to the transport model :

$$\begin{cases} \frac{d}{dt}X_i(t) &= \mathbf{v}\big(X_i(t),t\big) \qquad \forall\, t \in]t_i^0, t_i^f[\\ X_i(t_i^0) &= x_i^0 \end{cases} \tag{4}$$

where v is the transport velocity of the particles, t_0 and t_f are the time when the particle enters and leaves the domain. As a first estimate, the transport velocity v can be related to the shallow-water velocity u by a multiplicative constant c ($v = cu$). This coefficient c can be considered as unknown and identified in the optimization process. Then, we add the following term in the cost function expression: $\sum_{i=1}^{N} \int_{t_i^0}^{t_i^f} \left| X_i(t) - \bar{X}_i^{obs}(t) \right|^2 dt$.

Numerical experiments for an academic flow configuration and for a real canal flow demonstrate that this method makes it possible to significantly improve the identification either of inflow discharge or the local bed elevation and initial conditions, Fig. 2.

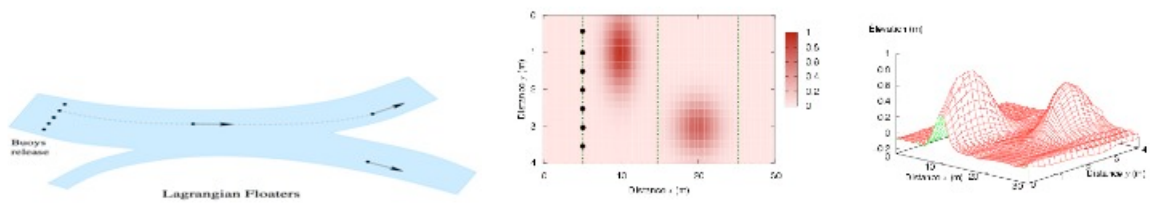

Fig. 2 a) Lagrangian floaters. b). Trajectories of surface particles. c) Topography identified.

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References

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