# A novel approach based on Voronoï cells to classify spectrogram zeros of multicomponent signals

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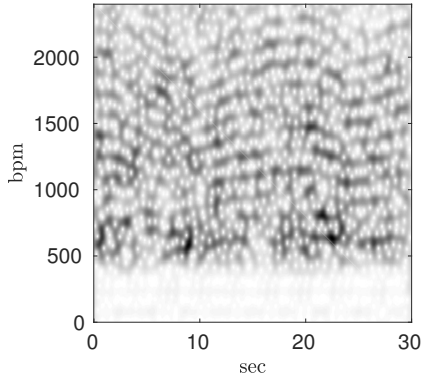
# Introduction

Identification of interference :

- Signal detection
- Signal reconstruction
- Localize separated modes

Fetal ECG signal



Short-time Fourier Transform

$$V_f^g(t,\eta) = \int_{\mathbb{R}} f(\tau) g(\tau - t) e^{-2i\pi(\tau - t)\eta} d\tau$$

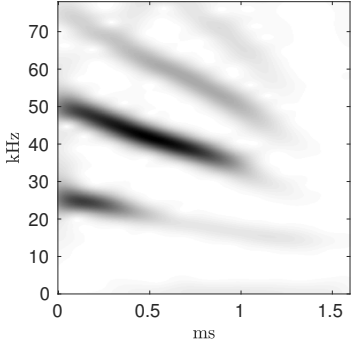
for $f, g \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$.

The *spectrogram* is $|V_f^g(t,\eta)|^2$

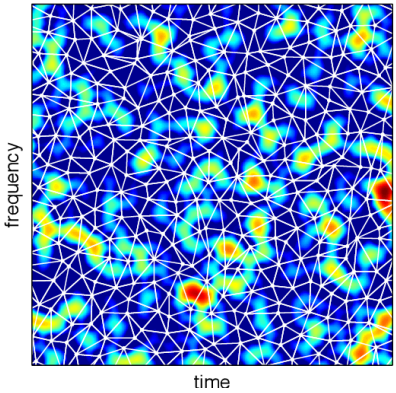
*Multicomponent signal*

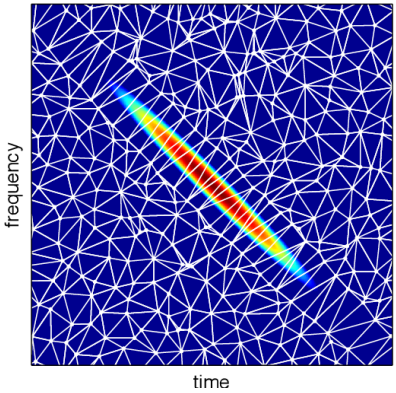
$$f(t) = \sum_{p=1}^{P} A_p(t) e^{2i\pi\phi_p(t)}$$

where $A_p(t) > 0$ and $\phi'(t) > 0$.

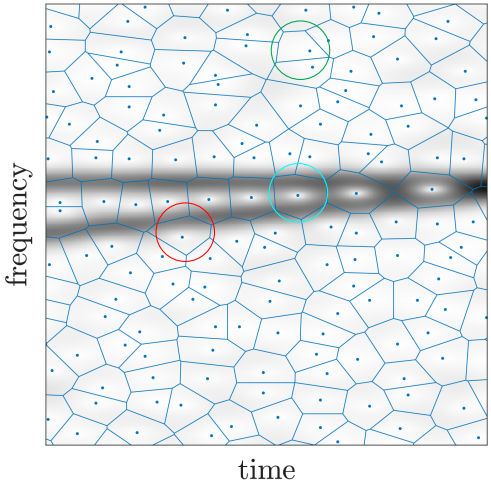


*Spectrogram zeros* : signal reconstruction [P. Flandrin]

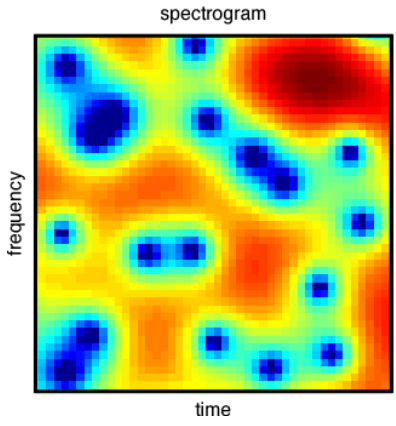


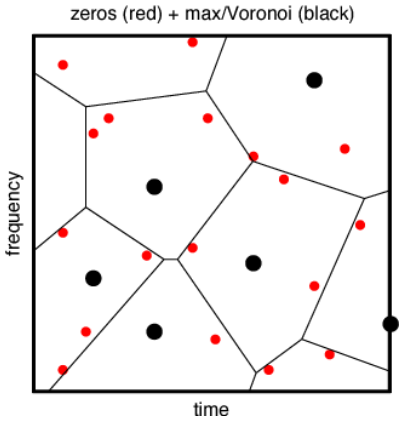



## *Spectrogram zeros* : interference



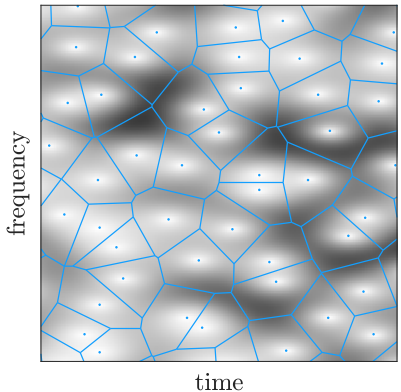
Study of *local maxima* [P. Flandrin]



# Study in pure noise

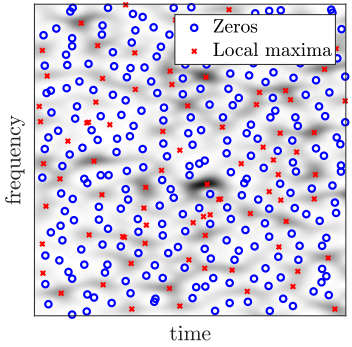
*Voronoï cell* $C_\varepsilon^i$ is a polygon.

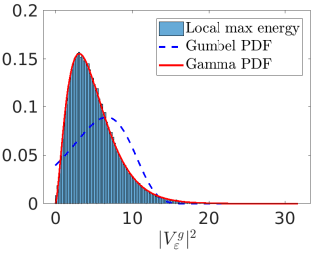


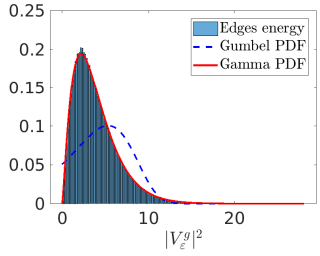

We study

$$\mathrm{Max}_\varepsilon^{i,j} = \max_{[n,k]\in C_\varepsilon^{i,j}} \frac{|V_\varepsilon^g[n,k]|^2}{\gamma^2},$$

where $\gamma := \sigma_\varepsilon \|g\|_2$.


Zeros
Local maxima
frequency
time


Local max energy
Gumbel PDF
Gamma PDF
0.2
0.15
0.1
0.05
0
0
10
20
30
$|V_\varepsilon^g|^2$


Edges energy
Gumbel PDF
Gamma PDF
0.25
0.2
0.15
0.1
0.05
0
0
10
20
$|V_\varepsilon^g|^2$

# Bias for noisy signals

*Biased* estimator

$$\widehat{\gamma} := \underset{[n,k]}{\text{median}} \frac{\left|\Re\mathfrak{e}\left\{V^g_{\tilde{f}}[n,k]\right\}\right|}{0.6745}$$

Proposed estimator

$$\tilde{\gamma} := \underset{[n,k]\in\mathcal{S}_{\tilde{f}}}{\text{median}} \frac{\left|\Re\mathfrak{e}\left\{V^g_{\tilde{f}}[n,k]\right\}\right|}{0.6745}, \qquad \text{where } \mathcal{S}_{\tilde{f}} = \{[n,k];\, |V^g_{\tilde{f}}[n,k]\| < 3\widehat{\gamma}\}.$$

We set a threshold $R$ from a probability of false alarm.

# Classification

Provided a *threshold* $R$ and a *voronoï cell* $C_f^i$ with $J^i$ edges

$$X_R^i = \#\{j \in [\![1, J^i]\!] : \mathrm{Max}_{\tilde{f}}^{i,j} > R\}$$

- **IF** $X_R^i = J^i$ : *signal-signal*
- **ELSE IF** $X_R^i \geq 3$ : *signal-noise*
- **ELSE** *noise-noise*

**Reference class for spectrogram zeros**

For a mode $f_p$

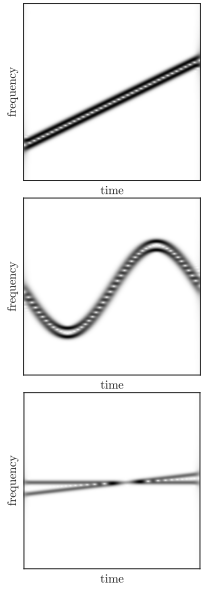
$$\mathcal{H}_{f_p} := \{[n,k]; |V_{f_p}^g[n,k]| > 3\gamma\}$$

For each SZ $z_i$, we verify intersections

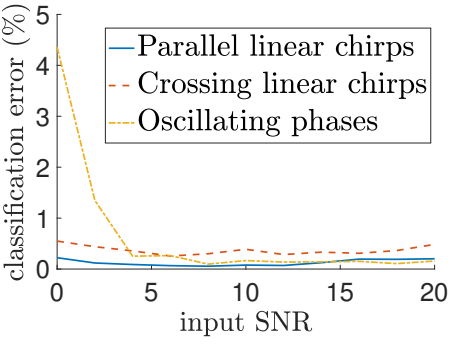
$$D^i := \mathrm{card}\{p \in [\![1,P]\!]; C_{\tilde{f}}^i \cap \mathcal{H}_{f_p} \neq \emptyset\}.$$

Reference classes

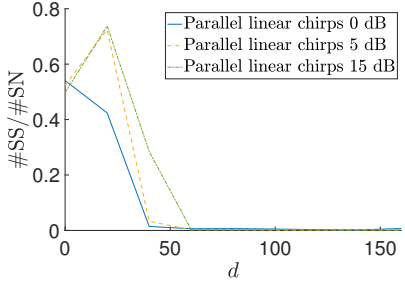
$$\mathcal{Z}_{SS} := \{z_i,\ D^i > 1\},$$
$$\mathcal{Z}_{SN} := \{z_i,\ D^i = 1\},$$
$$\mathcal{Z}_{NN} := \{z_i,\ D^i = 0\}.$$



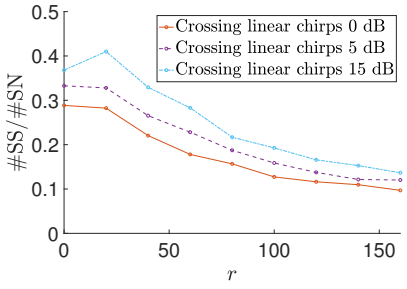

## Classification error



Indicator of separability

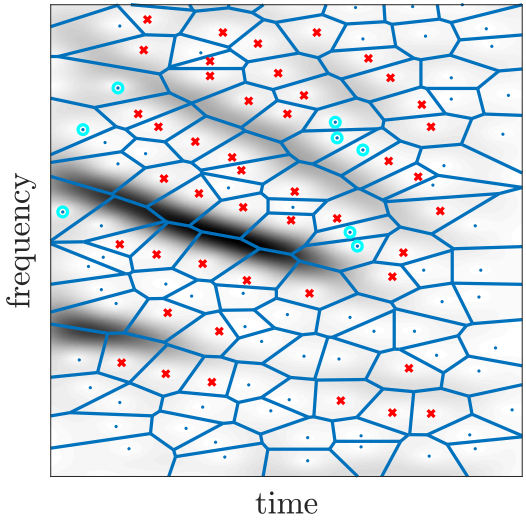



d : distance between the modes (frequency bins)




r : chirp rate of the second mode

Experiment on a real bat signal



Thank you for your attention