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Estimation of Overlapping Modes using Sparse Modeling of Signal Innovation

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Introduction

- Focus on multi-component signals (MCS).

$$x(n) = \sum_{k=0}^{K-1} x_k(n) \quad , \text{with } x_k(n) = a_k(n)\, e^{j\phi_k(n)}, \tag{1}$$

Investigated approaches

- Mixture of K superimposed components.
- $a_k(n)$ and $\phi_k(n)$ the time-varying amplitude and frequency of component k.

Problem statement

Objective

- Instantaneous frequency (IF) estimation.
- Overlapping components.
- Presence of noise.

Context of this work

- Time-frequency representation (TFR).
- Well design representation.
- Allows to observe the IF trajectory of each mode as a ridge.

Challenge

- Large variety of real signals (amplitude, modulation rate,. . .).
- Presence of external noise.
- Vicinity of ridges overlap.

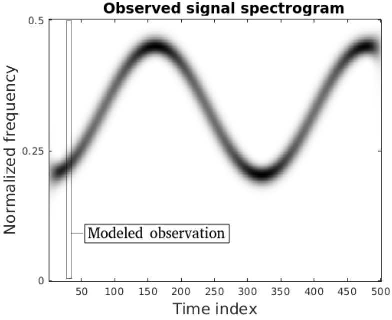
Plan

Plan

Observation model

Challenge

- Spectrogram of the observation: square modulus of the STFT.
- Model vertical signal spectrogram cut.
- 1D signal observed at a fixed time instant.

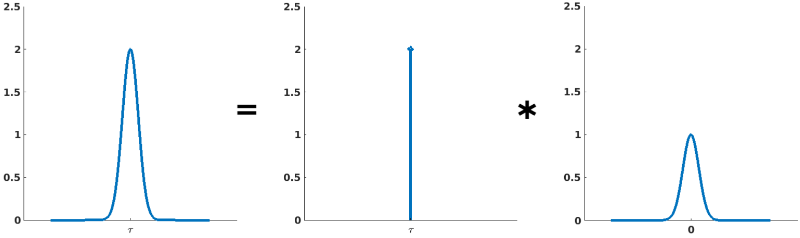


- $g(m) = e^{-\left(\frac{2\pi mL}{M}\right)^2}$.
- m : frequency in $[0, M-1]$.
- L : time spread of the analysis window

- Known Gaussian analysis window.
- Interest in its shift parameter.
- Indicates the position of each ridges.

Observation model

- Observation can be modeled as a discrete convolution between a stream of Dirac pulses and g.
- Known filter $g \rightarrow$ square of the Fourier transform of the analysis window.
- Pulses position (resp. weight) inform on the ridges location (resp. amplitude).



Observation model

- For a given stream of Dirac pulses associated with time instant n

$$f_n(m) = \sum_{k=0}^{K-1} a_k(n)\delta(m - \phi'_k(n)), \tag{2}$$

with $\phi'_k(n) = \frac{d\phi_k}{dn}(n)$ the IF, we observe

$$s_n(m) = \sum_{k=0}^{K-1} a_k(n)g(m - \phi'_k(n)). \tag{3}$$

- f_n is the most common example of signal with a finite rate of innovation (FRI).

Plan

Classical reconstruction

Sparse reconstruction

- Reconstructing FRI signal from a filtered and sampled version of them is a known problem[1].
- Let us write $\boldsymbol{s}_n$ as function of the inverse discrete time Fourier series (DTFS) of g.

$$\begin{aligned} \boldsymbol{s}_n(m) &= \sum_{k=0}^{K-1} a_k(n) \sum_{\lambda=-\infty}^{\infty} \hat{g}(\lambda)\, \mathrm{e}^{\frac{j2\pi\lambda(m-\phi_k'(n))}{M}} \\ &= \sum_{\lambda=-\infty}^{\infty} \hat{g}(\lambda) \underbrace{\sum_{k=0}^{K-1} a_k(n)\, \mathrm{e}^{\frac{-j2\pi\lambda\phi_k'(n)}{M}}}_{\hat{f}_n(\lambda)} \mathrm{e}^{\frac{j2\pi\lambda m}{M}}, \end{aligned} \tag{4}$$

with $\hat{g}$ (resp. $\hat{f}_n$) the Fourier transform of g (resp. f_n).

- Involve an infinite sum.

[1] M. Vetterli and P. Marziliano and T. Blu. Sampling signals with finite rate of innovation, 2002.

Classical reconstruction

Sparse reconstruction

- Bandlimited approximation: keep only $2M_0 + 1$ Fourier series coefficients[2].

$$\boldsymbol{s}_n(m) \approx \sum_{\lambda=-M_0}^{M_0} \hat{g}(\lambda)\hat{f}_n(\lambda)\,\mathrm{e}^{\frac{j2\pi\lambda m}{M}}. \tag{5}$$

- We can invert the matrix wise system

$$\boldsymbol{s}_n = \boldsymbol{V}\boldsymbol{D}_g\hat{\boldsymbol{f}}_n \Leftrightarrow \hat{\boldsymbol{f}}_n = \boldsymbol{D}_g^{-1}\boldsymbol{V}^{-1}\boldsymbol{s}_n, \tag{6}$$

- $\boldsymbol{V}$ is a Vandermonde matrix (Fourier atoms).
- $\boldsymbol{D}_g$ is diagonal and gathers the DTFS coefficients $\hat{g}(\lambda)$.

[2] A. Bhandari and A. M. Wallace and R. Raskar. Super-resolved time-of-flight sensing via FRI sampling theory, 2016.

Alternative reconstruction

Pulses location and weight recovery

- At this point, we should use the Prony method.
- Consists in computing a filter h that annihilates f_n.

$$\underbrace{\begin{pmatrix} \hat{f}_n(0) & \cdots & \hat{f}_n(-K+1) \\ \hat{f}_n(1) & \cdots & \hat{f}_n(-K+2) \\ \vdots & \vdots & \vdots \\ \hat{f}_n(K-1) & \cdots & \hat{f}_n(0) \end{pmatrix}}_{\boldsymbol{A}} \begin{pmatrix} h(1) \\ h(2) \\ \vdots \\ h(K) \end{pmatrix} = - \begin{pmatrix} \hat{f}_n(1) \\ \hat{f}_n(2) \\ \vdots \\ \hat{f}_n(K) \end{pmatrix} \qquad (7)$$

- The roots of the Z-transform of h are the $e^{\frac{-j\mathbf{2}\pi\phi'_k(n)}{M}}$.
- Prony method is limited in the presence of noise[1].

[1] M. Vetterli and P. Marziliano and T. Blu. Sampling signals with finite rate of innovation, 2002.

Alternative reconstruction

Presence of noise

- Inverting the Vandermonde matrix becomes an ill-posed problem.
- Computation of a filter h that does not annihilate f_n anymore.
- Existing alternatives.

Alternative

- Replacing the Prony method.
- Total Least-Squares alternative[3].
- Do not search anymore for a filter that annihilates f_n.
- Estimate a filter h minimizing $\|\boldsymbol{A}h\|^2$ under the constraint $\|h\|_2 = 1$.

[3] T. Blu and P. L. Dragotti and M. Vetterli and P. Marziliano and L. Coulot. Sparse sampling of signal innovations, 2008.

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Results

Experiments

- Comparison to a pseudo-Bayesian (PB) approach[4].
- Using several variational objectives (divergences, hyperparameters).
- Comparison of FRI (classical) and FRI_TLS (proposed).

$$\text{RMSE} = \sum_{k=0}^{K} \left[\sum_{n=0}^{N-1} \frac{\left(\bar{m}_{n,k} - \hat{m}_{n,k}\right)^2}{M^2} \right], \tag{8}$$

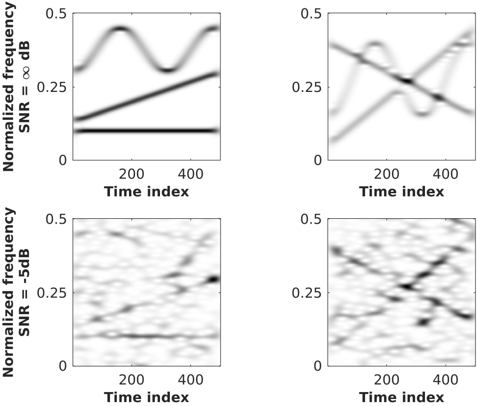
- $\bar{m}_{n,k}$ (resp. $\hat{m}_{n,k}$) the actual (resp. estimated) ridge position of the k-th component in the n-th time instant.
- White Gaussian noise with various Signal-to-noise ratio (SNR).

[4] Q. Legros, D. Fourer. A novel pseudo-Bayesian approach for robust multi-ridge detection and mode retrieval, 2020.

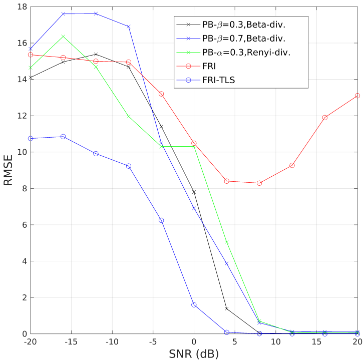
Results

Numerical experiments

- Reconstruction of two MCSs.
- With and without components overlapping.



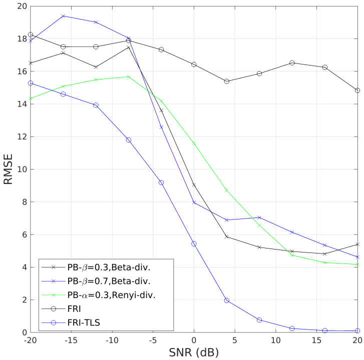
Results - without overlap



Observations

- Proposed FRI-TLS provides the best performance for all SNRs.
- FRI-TLS performs almost perfect estimation for SNR $> 0dB$.
- FRI not efficient for all SNRs.
- Robust to frequency modulation, involving broader ridges.

Results - with overlap



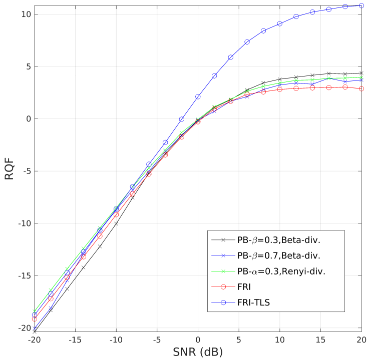
Observations

- Similar performance.
- FRI-TLS outperforms other approaches except at low SNR.
- Ability of estimating overlapping components (high SNR).

Results - with overlap

Experiments

- Reconstruction performance of an overlapping MCS.
- Reconstruction quality factor: $\text{RQF} = 10\log_{10}\left(\frac{||x||^2}{||x-\hat{x}||^2}\right)$.
- x (resp. $\hat{x}$) stands for the reference (resp. estimated) signal.
- Reconstruction using band-pass thresholding around the IF.



Observations

- Better reconstruction performance for SNR$> -5dB$ using FRI-TLS.
- Ability to deal with crossed ridges in noisy TFR.
- Robust methods perform better at low SNR due to strong noise in the IF vicinity.

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Conclusions and perspectives

Conclusions

- A novel approach for estimating the modes of a MCS in the presence of noise.
- Well suited to scenarios involving overlapping components.
- Improved performance compared to existing methods with and without overlap.

Future works

- Disentangle estimated components.
- Spatial regularization for cases involving the presence of perturbation around close ridges.

Thanks for your attention !

Codes available on my GitHub

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