ANR ASCETE : Partenaire Grenoble

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Outline

PhD thesis work: Nils Laurent (Oct 2019- Sep 2022)

Post-doc work: Neha Singh (Feb 2020-July 2021)

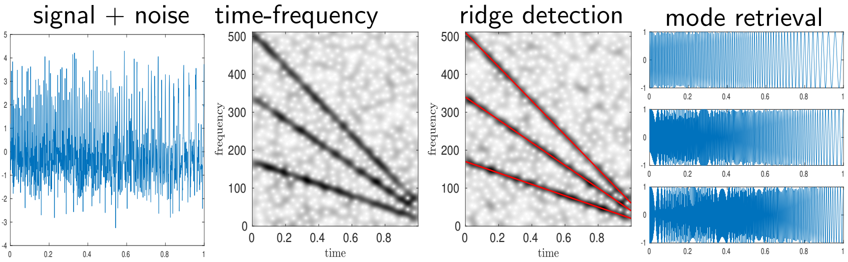
Inter-partners work

PhD thesis work: Nils Laurent (Oct 2019- Sep 2022)

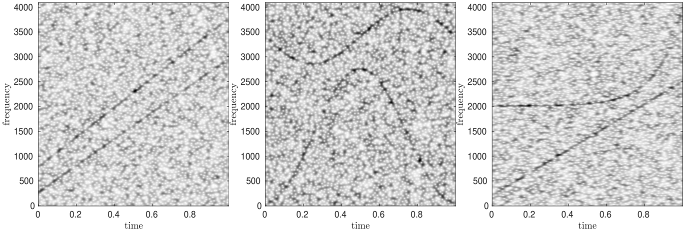
- New ridge extraction technique
- Improvement of chirp rate estimators (in collaboration with M. Colominas)

New ridge extraction technique

When f contains 3 modes:



Robust *ridge detection* and *reconstruction*



[1] N. Laurent and S. Meignen," A Novel Ridge Detector for Non Stationary Multicomponent Signals: Development and Application to Robust Mode Retrieval", IEEE TSP, vol. 69, pp. 3325-3336, 2021.

Discrete setting: $n \in \{1, \cdots, L\}$, $f[n] = f(\frac{n}{L})$

Noise : usually $\varepsilon[n]$ is an i.i.d. *complex white Gaussian noise*.

- $\Re\{\varepsilon\} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$
- $\Im\{\varepsilon\} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$

1. *Detection* of P ridges: $\Gamma_{p=1,\cdots,P}[n]$ on a grid of $M \times L$ *coefficients*

2. *Retrieval* of f_p based on Γ_p

- Signal + *white Gaussian noise* : $\tilde{f} = f + \varepsilon$

$$\hat{\gamma} = \frac{\mathrm{median} \left| \Re \left\{ V_{\tilde{f}}^{g}[n,k] \right\}_{n,k} \right|}{0.6745} \approx \sigma_{\varepsilon} \|g\|_2$$

- We consider coefficients above $\beta\hat{\gamma}$

$$\mathcal{S}(\beta) = \left\{ [n,k], |V_{\tilde{f}}^{g}[n,k]| \geq \beta\hat{\gamma} \right\}.$$

- Approach based on *LMMF*: $[n, m[n]]$ such that

$$|V_{\tilde{f}}^{g}[n,m[n]]| > |V_{\tilde{f}}^{g}[n,m[n]-1]| \quad \text{and} \quad |V_{\tilde{f}}^{g}[n,m[n]]| > |V_{\tilde{f}}^{g}[n,m[n]+1]|$$

- To construct the ridges, we need the definition of *reassignment operators*.

$$\tilde{t}_f(t,\eta) := t + \frac{V_f^{tg}(t,\eta)}{V_f^{g}(t,\eta)} \quad \text{and} \quad \tilde{\omega}_f(t,\eta) := \eta - \frac{1}{2i\pi}\frac{V_f^{g'}(t,\eta)}{V_f^{g}(t,\eta)}$$

  one sets $\widehat{\omega}_f(t,\eta) = \Re\{\tilde{\omega}_f(t,\eta)\}$ and $\widehat{t}_f(t,\eta) = \Re\{\tilde{t}_f(t,\eta)\}$.
  $f(t) = A(t)e^{2i\pi\phi(t)}$, if $A \in \mathbb{R}_0[X]$, $\phi \in \mathbb{R}_1[X]$, then $\widehat{\omega}_f(t,\eta) = \phi'(t)$.

- We also need an estimation of the *chirp rate*,

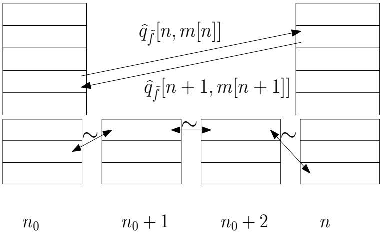
$$\tilde{q}_f(t,\eta) := \frac{\partial_t \tilde{\omega}_f(t,\eta)}{\partial_t \tilde{t}_f(t,\eta)},$$

  If $\log(A) \in \mathbb{R}_1[X]$, $\phi \in \mathbb{R}_2[X]$, $\widehat{q}_f(t,\eta) := \Re\{\tilde{q}_f\} = \phi''(t)$.

- Add *constraints* to relations between LMMF:

$[n, m[n]] \sim [n+1, m[n+1]]$

$[n_0, m[n_0]] \leftrightarrow [n, m[n]]$



- Then $\leftrightsquigarrow$ is defined like $\leftrightarrow$, but in $\mathcal{S}(\beta)$
- A *RRP* $\mathcal{R}_i$ is the finite set of LMMF sharing relation $\leftrightsquigarrow$.

- Connect ridges: *basins of attraction*

$$\mathcal{B}_i := \left\{ [n,k]; \quad \operatorname*{argmin}_{[x,y]\in \mathrm{RRP}} \left\| \left(\widehat{t}_{\tilde{f}}[n,k], \widehat{\omega}_{\tilde{f}}[n,k] \right) - [x,y] \right\| \in \mathcal{R}_i \right\}.$$

  It is a set of coefficients pointing to the $\mathcal{R}_i$.

- Definition of relevant basins:

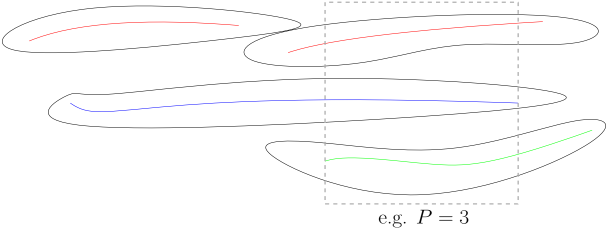
$$\mathcal{B}_i^{HT} = \begin{cases} \mathcal{B}_i \bigcap \mathcal{S}(2) & \text{if } \mathcal{R}_i \bigcap \mathcal{S}(3) \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}.$$

  $\mathcal{S}(2) \implies$ probability of *false alarm*: 10%.
  $\mathcal{S}(3) \implies$ probability of *false alarm*: 1%.

- Connected basins define *larger* time frequency regions denoted $\mathcal{C}_j^{HT}$.

Ranking P-tuples of $\{(\mathcal{C}_j^{HT})_j\}$ by coexistence: $(\{\mathcal{C}^{\kappa}_{p=1,\cdots,P}\})_{\kappa=0,\cdots,\kappa_{max}}$.

Illustration of $\mathcal{C}_1^{\kappa}, \mathcal{C}_2^{\kappa}, \mathcal{C}_3^{\kappa}$



Mode ridges : $\mathcal{A}_p^{\kappa} = (\mathcal{C}_p^0 \cup \mathcal{C}_p^1 \cup \cdots \cup \mathcal{C}_p^{\kappa}) \cap \mathcal{S}(3)$

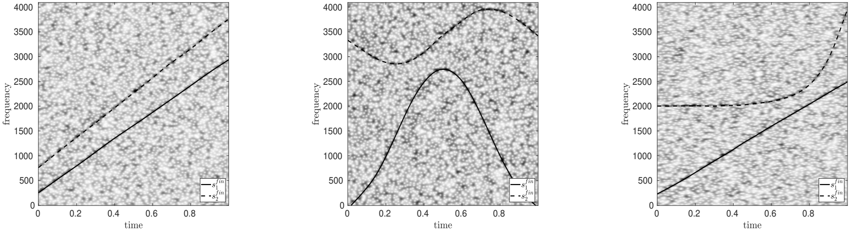
For $\kappa = 0, \cdots, \kappa_{max}$, we define $\mathcal{A}_p^\kappa = (\mathcal{C}_p^0 \cup \mathcal{C}_p^1 \cup \cdots \cup \mathcal{C}_p^\kappa) \cap \mathcal{S}(3)$ and then ridges as splines with *smoothness* λ:

$$\underbrace{s_p^\kappa}_{\text{cubic spline}} = \operatorname*{argmin}_s \Big[\underbrace{(1-\lambda) \sum_{[n,m[n]] \in \mathcal{A}_p^\kappa} \left(m[n]\frac{L}{M} - s(\frac{n}{L}) \right)^2 |V_{\tilde{f}}^g[n, m[n]]|}_{\text{data}} + \underbrace{\lambda \int_0^1 (s''(t))^2 dt}_{\text{regularity}} \Big].$$

The index of the solution is denoted by κ^{fin} and satisfies,

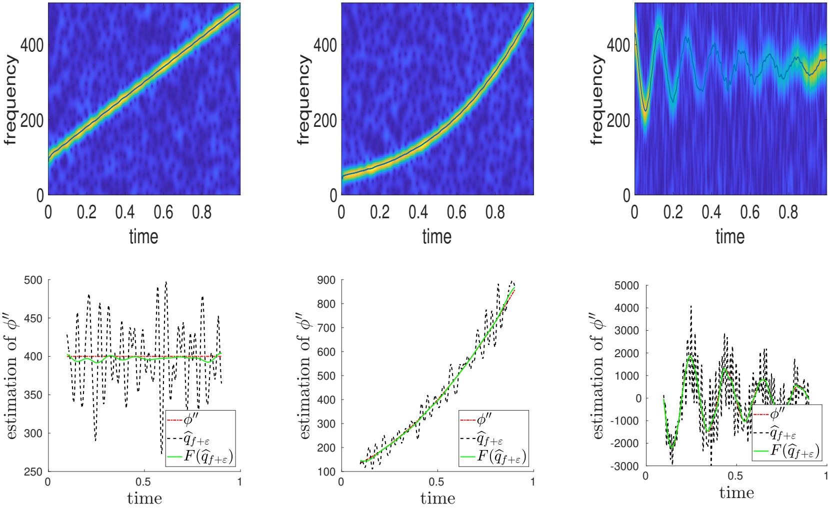
$$\kappa^{fin} = \operatorname*{arg\,max}_{\kappa \text{ s.t. } (s_p^\kappa)_p \textit{not crossing}} \sum_{p=1}^{P} E_p^\kappa,$$

where E_p^κ is an energy related to the spline s_p^κ.



frequency
time
s_1^{fin}
s_2^{fin}

Robust chirp rate estimation



[2] N. Laurent, S. Meignen and M. A. Colominas ," On Local Chirp Rate Estimation in Noisy Multicomponent Signals: With an Application to Mode Reconstruction", IEEE Transactions on Signal Processing, vol. 70, pp. 3429-3440, 2022.

- 2nd order estimation of the *chirp rate*

$$\widehat{q}_{f+\varepsilon} = -\frac{1}{2\pi}\Im\left\{\frac{(V^{g}_{f+\varepsilon})^2}{V^{g}_{f+\varepsilon}V^{t^2g}_{f+\varepsilon} - (V^{tg}_{f+\varepsilon})^2}\right\}$$

- Assuming f is a *linear chirp*, we simplify $\widehat{q}_{f+\varepsilon}$

$$\widehat{q}_{f+\varepsilon} \approx \widehat{q}_f + \underbrace{\frac{1}{2\pi}\Im\left\{\frac{V^{g}_{f}V^{t^2g}_{\varepsilon}}{(V^{t^2g}_{f})^2} - \frac{V^{g}_{\varepsilon}}{V^{t^2g}_{f}}\right\}}_{G}$$

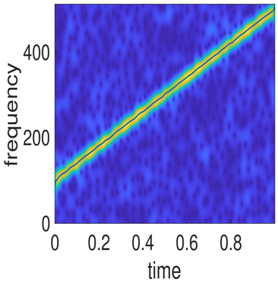
- $G(t)$ has the expression

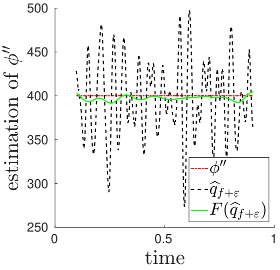
$$G(t) = -2\pi\phi''(t)\Re\left\{\frac{V^{t^2g}_{\varepsilon}}{V^{t^2g}_{f}} - \frac{V^{g}_{\varepsilon}}{V^{g}_{f}}\right\} + \frac{2\pi}{\sigma^2}\Im\left\{\frac{V^{t^2g}_{\varepsilon}}{V^{t^2g}_{f}} - \frac{V^{g}_{\varepsilon}}{V^{g}_{f}}\right\}$$

- To define a *low pass filter*, we study the *power spectral density* of G, with f a linear chirp with rate b,

$$P_G(\eta) = \frac{\sigma_\varepsilon^2 \sigma^6 4\pi^2 \eta^4}{(1 + b^2\sigma^4)^2} e^{-\frac{2\pi\sigma^2\eta^2}{1+b^2\sigma^4}}.$$

  It has its maximum at $\eta_m = \frac{\sqrt{1+b^2\sigma^4}}{\sigma\sqrt{\pi}}$.
- We set the *cut-off frequency* $\eta_{c,b}$ to a proportion of $P_G(\eta_m)$ and define a filtered estimate $F(\widehat{q}_{f+\varepsilon})$, assuming $b = 0$.





Study of reassignment operators

Four different objectives

- To introduce a novel matricial form for the derivation of IF estimators used in FSSTs.
- To characterize the zeros of the reassignment vectors associated with different types of FSSTs.
- To investigate reassignment vectors in the case of interfering pure harmonic modes and of noisy linear chirps.
- To propose a new IF estimator based on the determination of relevant points extracted from FSSTs ridges.

[3] S. Meignen and N. Singh, "Analysis of Reassignment Operators Used in Synchrosqueezing Transforms: with an Application to Instantaneous Frequency Estimation", IEEE Transactions on Signal Processing, vol. 70, pp.216-227, 2021.

We consider $f(\tau) = A(\tau)e^{i2\pi\phi(\tau)}$ with $\log(A(\tau))$ (*resp.* $\phi(\tau)$) a polynomial of order S (*resp.* N) for τ close to t, with $S \leq N$, namely:

$$f(\tau) = \exp\left(\sum_{j=0}^{N} \frac{\left([\log(A)]^{(j)}(t) + i2\pi\phi^{(j)}(t)\right)(\tau - t)^j}{j!}\right). \qquad (1)$$

From (1), and the definition of STFT we may write:

$$\partial_t V_f^h(t,\eta) = \quad r_1^{[N]}(t)V_f^h(t,\eta) + \sum_{j=2}^{N} r_j^{[N]}(t)V_f^{t^{j-1}h}(t,\eta) \qquad (2)$$

where $r_j^{[N]}(t) = \frac{[\log(A)]^{(j)}(t)+2i\pi\phi^{(j)}(t)}{(j-1)!}$.

- When f is a MCS, the equality (2) turns into an approximation, namely for (t,η) in the vicinity of $(t,\phi_k'(t))$ for some k, one may write:

$$\partial_t V_f^h(t,\eta) = r_1^{[N]}(t,\eta) V_f^h(t,\eta) + \sum_{j=2}^{N} r_j^{[N]}(t,\eta) V_f^{t^{j-1}h}(t,\eta), \qquad (3)$$

where $r_j^{[N]}(t,\eta) \approx \frac{[\log(A_k)]^{(j)}(t)+2i\pi\phi_k^{(j)}(t)}{(j-1)!}$.

- In that context, $\widehat{\omega}_f^{[N]}(t,\eta) := \Re\left\{\frac{r_1^{[N]}(t,\eta)}{2i\pi}\right\}$ is the Nth order LIF estimator of f_k.

A simple way to compute $r_1^{[N]}$ is to consider Eq. (3) and to remark that $\partial_\eta V_f^h(t,\eta) = -2i\pi V_f^{th}(t,\eta)$, can be written under the matrix form:

$$\begin{bmatrix} \partial_t V_f^h \\ \frac{i}{2\pi}\partial_\eta \partial_t V_f^h \\ \vdots \\ \frac{i^{N-1}}{(2\pi)^{N-1}}\partial_\eta^{N-1}\partial_t V_f^h \end{bmatrix} = \begin{bmatrix} V_f^h & V_f^{th} & \cdots & V_f^{t^{N-1}h} \\ V_f^{th} & V_f^{t^2 h} & \cdots & V_f^{t^N h} \\ \vdots & \vdots & \ddots & \vdots \\ V_f^{t^{N-1}h} & V_f^{t^N h} & \cdots & V_f^{t^{2(N-1)}h} \end{bmatrix} \begin{bmatrix} r_1^{[N]} \\ r_2^{[N]} \\ \vdots \\ r_N^{[N]} \end{bmatrix} = DR. \quad (4)$$

Based on simple properties of the determinant of matrices, one obtains that:

$$r_1^{[N]} = \frac{det(M_1)}{det(D)}, \tag{5}$$

with

$$M_1 = \begin{bmatrix} \partial_t V_f^h & V_f^{th} & \cdots & V_f^{t^{N-1}h} \\ \frac{i}{2\pi}\partial_\eta \partial_t V_f^h & V_f^{t^2h} & \cdots & V_f^{t^N h} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{i^{N-1}}{(2\pi)^{N-1}}\partial_\eta^{N-1}\partial_t V_f^h & V_f^{t^N h} & \cdots & V_f^{t^{2(N-1)}h} \end{bmatrix}. \tag{6}$$

Then, as $\partial_t V_f^h = i2\pi\eta V_f^h - V_f^{h'}$, one gets, for any $l \geq 1$:

$$\partial_\eta^l \partial_t V_f^h = (-2i\pi)^l \left(-kV_f^{t^{l-1}h} - V_f^{t^l h'} + 2i\pi\eta V_f^{t^l h} \right), \tag{7}$$

- This leads to: $det(M_1) = i2\pi\eta det(D) - det(U_1) - det(V_1)$ with:

$$U_1 = \begin{bmatrix} 0 & V_f^{th} & \cdots & V_f^{t^{N-1}h} \\ V_f^{h} & V_f^{t^2h} & \cdots & V_f^{t^Nh} \\ \vdots & \vdots & \ddots & \vdots \\ (N-1)V_f^{t^{N-2}h} & V_f^{t^Nh} & \cdots & V_f^{t^{2(N-1)}h} \end{bmatrix}, \quad V_1 = \begin{bmatrix} V_f^{h'} & V_f^{th} & \cdots & V_f^{t^{N-1}h} \\ V_f^{th'} & V_f^{t^2h} & \cdots & V_f^{t^Nh} \\ \vdots & \vdots & \ddots & \vdots \\ V_f^{t^{N-1}h'} & V_f^{t^Nh} & \cdots & V_f^{t^{2(N-1)}h} \end{bmatrix} \tag{8}$$

and thus

$$\widehat{\omega}_f^{[N]} = \frac{\Im\left\{r_1^{[N]}\right\}}{2\pi} = \eta - \frac{1}{2\pi}\Im\left\{\frac{det(U_1) + det(V_1)}{det(D)}\right\}. \tag{9}$$

- Gaussian window : $h'(t) = -\frac{2\pi}{\sigma^2}th(t)$, the first two columns of V_1 are colinear and its determinant is null. In that context, one may thus write:

$$\widehat{\omega}_f^{[N]} = \frac{1}{2\pi}\Im\left\{r_1^{[N]}\right\} = \eta - \frac{1}{2\pi}\Im\left\{\frac{det(U_1)}{det(D)}\right\}. \tag{10}$$

Characterization of the Zeros of Reassignment Vectors

- When $N = 1$:

$$\widehat{\omega}_f - \eta = \Im\left\{\frac{1}{\sigma^2}\frac{V_f^{th}}{V_f^h}\right\} = -\Im\left\{\frac{\partial_\eta V_f^h}{2i\pi\sigma^2 V_f^h}\right\} = \frac{1}{4\pi\sigma^2}\frac{\partial_\eta |V_f^h|^2}{|V_f^h|^2}, \quad (11)$$

zeros are points (t,η) such that $\partial_\eta |V_f^h(t,\eta)|^2 = 0$.

- When $N = 2$:

$$\widehat{\omega}_f^{[2]} - \eta = \quad \Im\left\{\frac{1}{2\pi}\frac{V_f^h V_f^{th}}{V_f^h V_f^{t^2h} - (V_f^{th})^2}\right\} \quad (12)$$

that is to say

$$0 = \Im\left\{V_f^h V_f^{th}(V_f^h V_f^{t^2h} - (V_f^{th})^2)^*\right\} = |V_f^h|^2 \partial_\eta |V_f^{th}|^2 - |V_f^{th}|^2 \partial_\eta |V_f^h|^2.$$

which can also be viewed as

$$det \begin{bmatrix} |V_f^h|^2 & \partial_\eta |V_f^h|^2 \\ |V_f^{th}|^2 & \partial_\eta |V_f^{th}|^2 \end{bmatrix} = 0.$$

The reassignment vector when $N = 2$ thus reads:

$$\widehat{\omega}_f^{[2]} - \eta = \frac{|V_f^h|^2 \partial_\eta |V_f^{th}|^2 - |V_f^{th}|^2 \partial_\eta |V_f^h|^2}{|V_f^h V_f^{t^2 h} - (V_f^{th})^2|^2}. \quad (13)$$

Approximating reassignment vectors in the vicinity of their zeros

- We first approximate second order reassignment vector, i.e. $N = 2$, considering that, in the vicinity of its zeros, V_f^{th} is small (for a linear chirp it is null on the ridge).
- It is then natural to consider the following approximation of $\widehat{\omega}_f^{[2]}(t,\eta) - \eta$ in the vicinity of its zeros:

$$\widehat{\omega}_f^{[2]}(t,\eta) - \eta = \Im\left\{\frac{1}{2\pi}\frac{V_f^h V_f^{th}}{V_f^h V_f^{t^2h} - (V_f^{th})^2}\right\} = \Im\left\{\frac{1}{2\pi}\frac{V_f^{th}}{V_f^{t^2h}}\frac{1}{1 - \frac{(V_f^{th})^2}{V_f^h V_f^{t^2h}}}\right\}$$
$$\approx \Im\left\{\frac{1}{2\pi}\frac{V_f^{th}}{V_f^{t^2h}}\right\} + \Im\left\{\frac{1}{2\pi}\frac{(V_f^{th})^3}{V_f^h (V_f^{t^2h})^2}\right\}.$$

- When $\frac{(V_f^{th})^2}{V_f^h V_f^{t^2h}} << 1$, one can approximate $\widehat{\omega}_f^{[2]}$ only including the first order term in V_f^{th}, it is denoted by $\widehat{\omega}_{f,1}^{[2]}$. When one considers two terms in the approximation, we denote it by $\widehat{\omega}_{f,2}^{[2]}$.

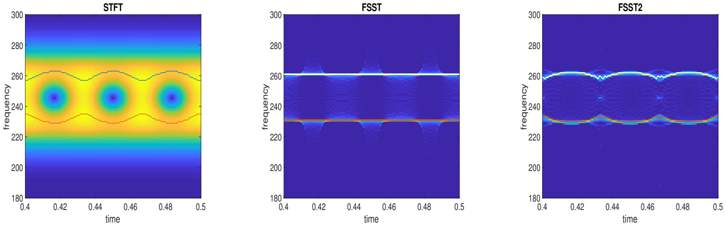
Study of the reassignment vector on interfering modes

- Let us consider that $f(t) = f_1(t) + f_2(t)$ with $f_1(t) = Ae^{i2\pi\xi_1 t}$ and $f_2(t) = e^{i2\pi\xi_2 t}$, where $\xi_1 < \xi_2$.
- When h is the Gaussian window:

$$
\begin{aligned}
V_{f_1}^h(t,\eta) &= \hat{h}(\eta-\xi_1)Ae^{i2\pi\xi_1 t} = \sigma Ae^{i2\pi\xi_1 t}e^{-\pi(\eta-\xi_1)^2\sigma^2} \\
V_{f_2}^h(t,\eta) &= \sigma e^{i2\pi\xi_2 t}e^{-\pi(\eta-\xi_2)^2\sigma^2} \\
|V_f^h(t,\eta)|^2 &= \sigma^2(A^2e^{-2\pi\sigma^2(\eta-\xi_1)^2} + e^{-2\pi\sigma^2(\eta-\xi_2)^2} \\
&\quad +2Ae^{-\pi\sigma^2\left[(\eta-\xi_1)^2+(\eta-\xi_2)^2\right]}\cos(2\pi(\xi_2-\xi_1)t))
\end{aligned}
$$

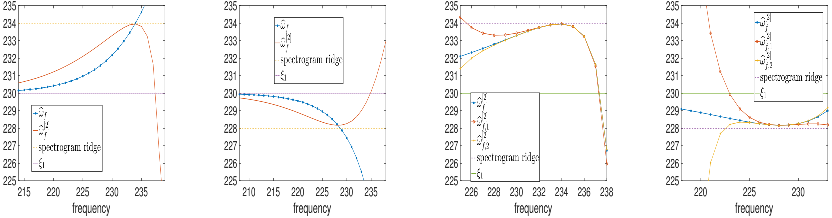
t_k and $\tilde{t}_k$ correspond to minima of the spectrogram with respect to η

Illustration of different behaviors



(a) (b) (c)

(a): STFT of two interfering modes, with the two ridges associated with local maxima superimposed; (b): FSST of the signal in (a); (c): FSST2 of the signal in (a)



(a) (b) (c) (d)

(a): $\widehat{\omega}_f$ and $\widehat{\omega}_f^{[2]}$ in the vicinity of the lower spectrogram ridge at time t_k; (b): same as (a) but at time $\tilde{t}_k$; (c): $\widehat{\omega}_f^{[2]}$, $\widehat{\omega}_{f,1}^{[2]}$, and $\widehat{\omega}_{f,2}^{[2]}$ in the vicinity of the lower spectrogram ridge at time t_k; (d): same as (c) but at time $\tilde{t}_k$.

Mathematical analysis

Proposition

On the upper (resp. lower) spectrogram ridge the second order reassignment vector is oriented towards higher (resp. lower) frequencies except at time instants t_k and $\tilde{t}_k$.

- The TF coefficients are not reassigned onto the spectrogram ridges with FSST2: the point on the upper (resp. lower) spectrogram ridge (except those at time t_k and $\tilde{t}_k$) are reassigned at a higher (resp. lower) frequency.
- The spectrogram ridges are the zeros of the first order reassignment vector but are very different from *FSST ridges*.

Inter-partners work

- Work on the zeros of the spectrogram (joint work with Nantes partner)
- Work on phase retrieval comparison of PGHI and Griffin-Lim algorithms (joint work with Paris, internship in 2022)
- Potential common interest in Deep Learning approach (with Paris and university of Lund, Sweden)

Thanks for you attention!