Projet ANR ASCETE

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Last work on *ridge detection*:

- With a high noise level,
- Simultaneous detection for multiple modes,
- Work is published in IEEE TSP.

Current work on *ECG signals*:

- Extraction of the fetal signal in ECG : When QRS peaks are difficult to detect, TF analysis is a powerfull technique,
- Analysis of the NMF results,
- How to link the harmonics of the fetal signal.

Table of Contents

Time frequency analysis

The *multicomponent signal* (MCS) f of length L is defined as:

$$f[n] = \sum_{p=1}^{P} f_p[n] \quad \text{with} \quad f_p[n] = A_p[n]e^{i2\pi\phi_p[n]}, \tag{1}$$

where $P \in \mathbb{N}$, $A_p[n]$ and $\phi'_p[n]$ being respectively the *instantaneous amplitude* (IA) and *instantaneous frequency* (IF) of f_p. The noisy signal is expressed by:

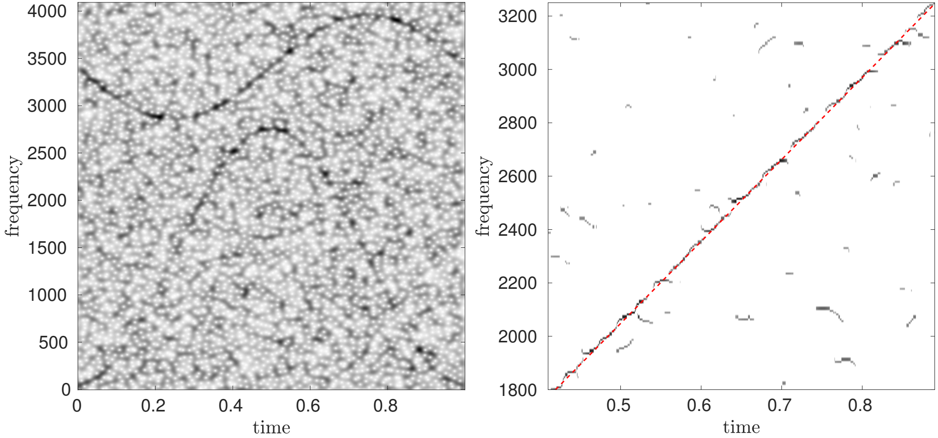
$$\tilde{f} := f + \varepsilon \tag{2}$$

An the STFT of $\tilde{f}$ with a window g is defined as follows:

$$V_{\tilde{f}}^{g}[m,k] := \sum_{n=0}^{N-1} \tilde{f}[n+m-M]g[n-M]e^{-2i\pi\frac{k}{N}(n-M)}, \tag{3}$$

High noise level

Following the ridge raises problems when the noise creates *dicontinuities*:



To create a *ridge portion*, *local modulus maxima along frequency axis* (LMMF) are linked using the operator $\hat{q}_{\tilde{f}}$:

$$\hat{q}_{\tilde{f}}[n,k] := \Re\left\{\frac{1}{2i\pi}\frac{V_{\tilde{f}}^{g''}[n,k]V_{\tilde{f}}^{g}[n,k]-(V_{\tilde{f}}^{g'}[n,k])^2}{V_{\tilde{f}}^{tg}[n,k]V_{\tilde{f}}^{g'}[n,k]-V_{\tilde{f}}^{tg'}[n,k]V_{\tilde{f}}^{g}[n,k]}\right\}, \tag{4}$$

Then, we link generic LMMFs named $m[n]$ with below relation:

$$([n,m[n]] \sim [n+1,m[n+1]]) \Leftrightarrow$$
$$\begin{cases} m[n+1] := \underset{k}{\operatorname{argmin}}\left\{\left|k-m[n]-\hat{q}_{\tilde{f}}[n,m[n]]\frac{N}{L^2}\right|, \text{s.t. } [n+1,k] \text{ LMMF}\right\} \\ m[n] := \underset{k}{\operatorname{argmin}}\left\{\left|m[n+1]-k-\hat{q}_{\tilde{f}}[n+1,m[n+1]]\frac{N}{L^2}\right|, \text{s.t. } [n,k] \text{ LMMF}\right\}. \end{cases} \tag{5}$$

In other words, when $m[n] \sim m[n+1]$, then $q[n,m[n]]$ points towards $m[n+1]$ and vice versa.

This definition is extended in the case LMMFs are not neighbors:

$$([n_0, m[n_0]] \leftrightarrow [n, m[n]]) \Leftrightarrow \begin{cases} \exists \ [n_0+1, m[n_0+1]], ..., [n-1, m[n-1]] \ \text{LMMFs} \\ \forall i = n_0, \cdots, n-1, \ [i, m[i]] \sim [i+1, m[i+1]] \end{cases} \quad (6)$$

A RP $\mathcal{R}$ containing LMMF $[n_0, m[n_0]]$, is finally defined by:

$$\mathcal{R}[n_0, m[n_0]] = \{[n, m[n]], \text{s.t. } [n, m[n]] \leftrightarrow [n_0, m[n_0]]\}. \quad (7)$$

Note that, if $[n, m[n]] \in \mathcal{R}[n_0, m[n_0]]$ then $\mathcal{R}[n, m[n]] = \mathcal{R}[n_0, m[n_0]]$.

When ε is a Gaussian white noise with variance σ_ε^2, below expression is an estimator of $\sigma_\varepsilon \|g\|_2$:

$$\hat{\gamma} = \frac{\mathrm{median}\left|\Re\left\{V_{\tilde{f}}^{g}[n,k]\right\}_{n,k}\right|}{0.6745}, \tag{8}$$

by considering

$$\mathcal{S}(\beta) = \left\{[n,k], |V_{\tilde{f}}^{g}[n,k]| \geq \beta\hat{\gamma}\right\}, \tag{9}$$

the chaining of LMMFs can be restricted to *relevant coefficients*,

$$[n_0, m[n_0]] \leftrightsquigarrow [n, m[n]] \Leftrightarrow$$
$$\begin{cases} \exists\ [n_0+1, m[n_0+1]], \cdots, [n-1, m[n-1]]\ \mathrm{LMMFs} \\ \forall i = n_0, \cdots, n-1, \begin{cases} [i, m[i]] \sim [i+1, m[i+1]] \\ [i, m[i]] \in \mathcal{S}(\beta) \end{cases} \end{cases} \tag{10}$$

the ridge portions defined using (10) gives the *relevant ridge portion* (RRP).

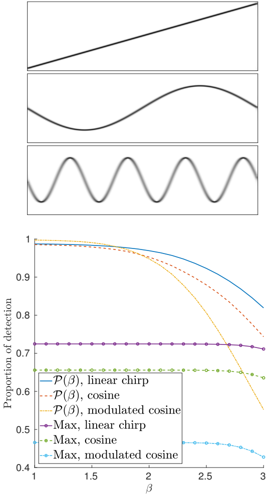
Setting parameter β

The study of the effect of β on the number of LMMF around the ridge requires to use ϕ' and ϕ''. It is then possible to define $\mathcal{I}$ as the vicinity of the IF:

$$\Delta_{LC}[n] = \frac{1}{\sqrt{2\pi}\sigma}\sqrt{1+\sigma^4\phi''[n]^2},$$
$$\mathcal{I}[n] = [(\phi'[n]-\Delta_{LC}[n])\frac{N}{L},(\phi'[n]+\Delta_{LC}[n])\frac{N}{L}], \quad (11)$$

at -10dB, setting $\beta = 2$ allows to remove part of the LMMFs related to noise, while keeping those in the vicinity of the IF:

$$\mathcal{P}(\beta) = \frac{\#\{[n,m[n]] \in \mathcal{S}(\beta) \text{ with } m[n] \in \mathcal{I}[n]\}}{L}, \quad (12)$$



Identify TF regions

Recalling reassignement vectors definition:

$$\hat{\omega}[n,k] = k\frac{L}{N} - \Re\left\{\frac{1}{2i\pi}\frac{V_f^{g'}[n,k]}{V_f^{g}[n,k]}\right\}, \qquad \hat{\tau}[n,k] = \frac{n}{L} + \Re\left\{\frac{V_f^{tg}[n,k]}{V_f^{g}[n,k]}\right\}, \tag{13}$$

A *basin of attraction* is defined as $\mathcal{B}_i := \{(n,k) \text{ s.t. } (\hat{\omega},\hat{\tau}) \text{ points to the same } \mathcal{R}_i\}$. The following restriction is applied:

$$\mathcal{B}_i^{HT} = \begin{cases} \mathcal{B}_i \bigcap \mathcal{S}(2) & \text{if } \mathcal{R}_i \bigcap \mathcal{S}(3) \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}, \tag{14}$$

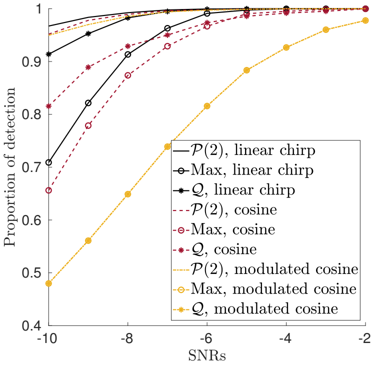
By gathering together *connected* $\mathcal{B}_i^{HT}$s in the TF plane, one obtains a set of larger TF regions which are denoted by $\{(\mathcal{C}_j^{HT})_j\}$.

With the respective definition the *ridge and region energies*:

$$E(\mathcal{R}_i) = \sum_{[n,k]\in\mathcal{R}_i \bigcap \mathcal{S}(3)} |V_{\tilde{f}}^{g}[n,k]|, \quad E(\mathcal{C}_j^{HT}) = \sum_{i,\mathcal{R}_i\subset\mathcal{C}_j^{HT}} E(\mathcal{R}_i), \tag{15}$$

the maximum of $E[n,m[n]] = E(\mathcal{C}_j^{HT})$ is more often in the vicinity of the IF than the maximum of the spectrogram:

$$\mathcal{Q} = \frac{\#\left\{n, \max_k(E[n,k]) > 0 \text{ and } (\operatorname{argmax}_k E[n,k]) \in \mathcal{I}[n]\right\}}{L}, \tag{16}$$



Weighted spline approximation

The iterative algorithm rank P-tuples of $\{(\mathcal{C}_j^{HT})_j\}$ that *coexist on the longest set of time indices* denoted by the sequences $(\mathcal{C}^0_{p=1,\cdots,P}),(\mathcal{C}^1_{p=1,\cdots,P}),\cdots$. Then, by defining the sequence:

$$\mathcal{A}_p^0=\mathcal{C}_p^0\bigcap\mathcal{S}(3),\;\;\mathcal{A}_p^1=(\mathcal{C}_p^0\bigcup\mathcal{C}_p^1)\bigcap\mathcal{S}(3),\quad\cdots \tag{17}$$

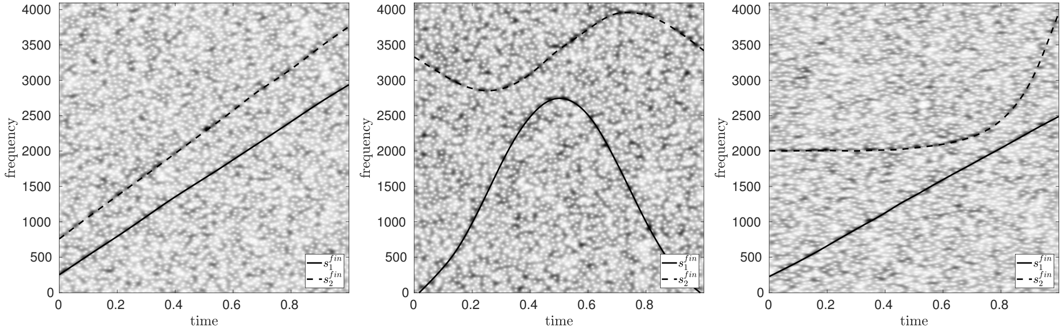
we compute the following *weighted spline approximation* for each mode:

$$s_p^q=\operatorname*{argmin}_s\left[(1-\lambda)\sum_{[n,m[n]]\in\mathcal{A}_p^q}|m[n]\frac{L}{N}-s(\frac{n}{L})|^2|V_{\tilde{f}}^g[n,m[n]]|+\lambda\int_0^1(s''(t))^2dt,\right]. \tag{18}$$

After some iterations, the procedure stabilizes leading to the final spline.

Solution for the ridge detection

The resulting splines are depicted on top of the noisy STFT in the following figures:



Some comparisons with other detectors

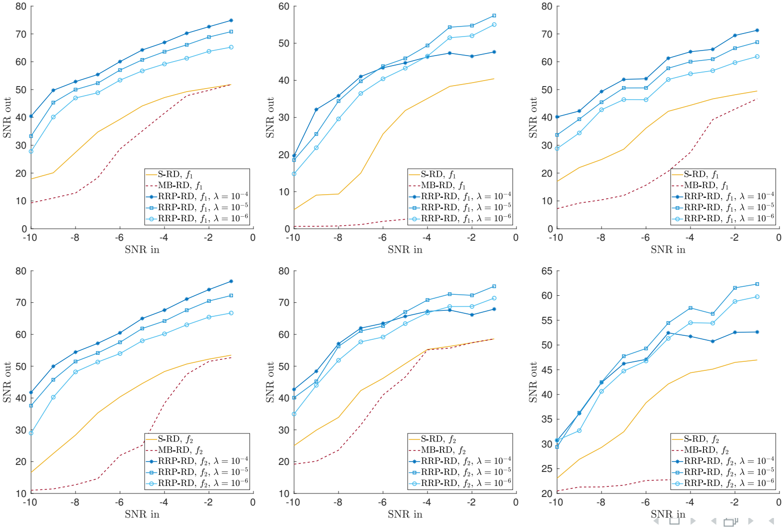
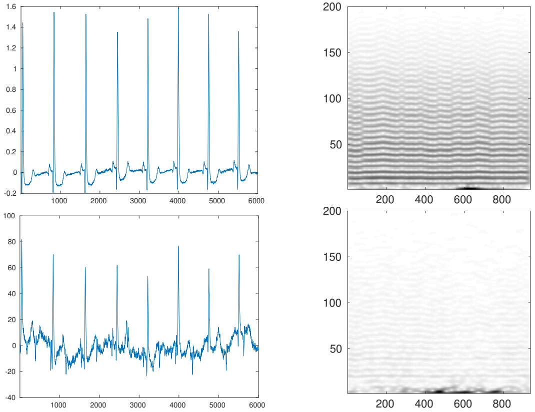


Table of Contents

Recorded ECG signals

Abdominal and thoracic signals:



Time frequency analysis is often considered when dealing with:

- detection of sleep apnea,
- study of arrhythmia,
- identification of coronary artery deseases ,
- fetal signal extraction.

Model of measured ECG signal

Defining the sum of pure harmonic modes:

$$h(t) = \sum_{p=1}^{P} A_p(t) e^{2i\pi p\phi(t)} \tag{19}$$

the measured ECG signal x is given by:

$$x(t) = f(t) + m(t) + \varepsilon(t), \tag{20}$$

where f and m are of type (19) and ε represents all possible noises: baseline wander, power line interference, fetal movements, ...
Computation of fetal HR is done in two steps, first by removing mother signal using adaptive filtering and then by estimating HR for which there exist many different techniques.

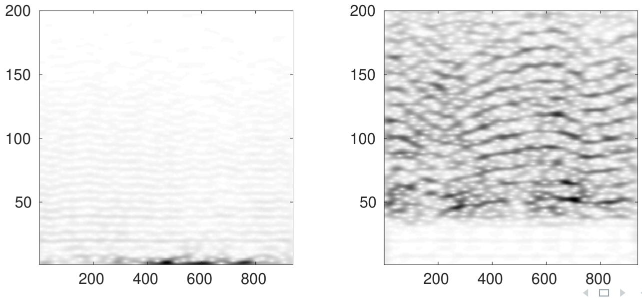
Preprocessing using adaptive filtering

The preprocessing consists of the two following operations:

1. *high pass* and *low pass* filtering in order to remove measurment noise,
2. *nonlinear adaptive filtering* is applied to *reduce* the power of the *mother signal*.

We name y the signal resulting by preprocessing x.

Raw abdominal ECG and preprocessed abdominal ECG:



NMF based approximation

The *non negative matrix factorization* (NMF) consiste of approximating a non negative matrix $X \in \mathcal{M}_{N,L}(\mathbb{R})$ with the product of two other matrices W, H and a number of component K:

$$\text{find } W \in \mathcal{M}_{N,K}(\mathbb{R}),\ H \in \mathcal{M}_{K,L}(\mathbb{R}) \text{ such that } X \approx WH \text{ and } W, H \geq 0 \tag{21}$$

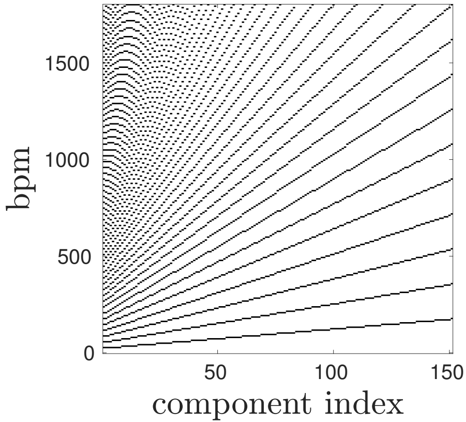
In our case, the matrix X is a *time frequency reprsentation* (TFR) of the preprocessed signal y. In the ideal case, by expressing the constraints in W, solving:

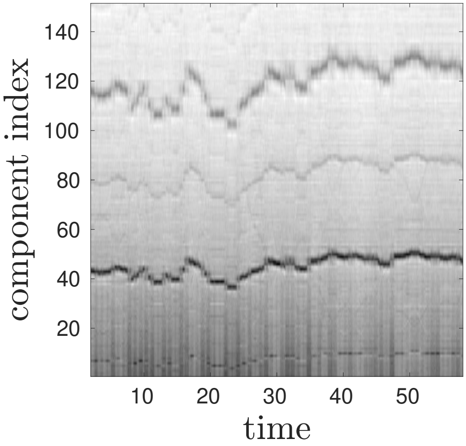
$$\text{find } H \in \mathcal{M}_{K,L}(\mathbb{R}) \text{ such that } X \approx WH \text{ and } H \geq 0 \tag{22}$$

would mean that the minimal component in H correspond to the fetal HR.

Example of NMF decomposition

Example of decomposition in *good conditions*:





NMF based approximation

Because the signal y may not respect the *harmonic model*, such a decomposition is not safe. To reduce the complexity, one can *separate harmonics and envelope estimation*, leading to below expression:

$$X = X_{(e)} \odot X_{(\varphi)} \approx (W_{(e)}H_{(e)}) \odot (W_{(\varphi)}H_{(\varphi)}) \tag{23}$$

by expressing harmonicity constraint in a fixed dictionnary $W_{(e)}$ and setting a certain degree of freedom for the envoloppe in $W_{(\varphi)}$. The estimation of the HR can be done by taking the minimum of $H_{(e)}$.

The selection of the minimum of $H_{(e)}$ does not ensure good HR estimation. For instance, wrong results appear in below cases:

- When the signal is composed of *multiple sources*.
- When the signal does not respect the *harmonic model*.

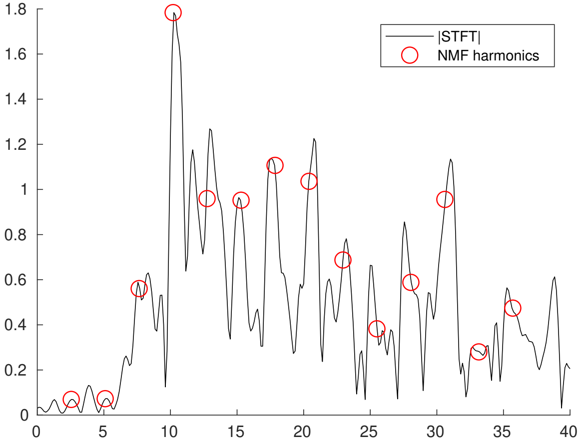
In those cases, $W_{(e)}$ is not an appropriate dictionnary, because y is far from the model. As a consequence, the NMF algorithm will then try to minimise the error by doing *combination of componenents* in $H_{(e)}$.

Furthermore, as $H_{(e)}$ and $H_{(\varphi)}$ are initialized randomly, it makes the analysis of this technique more difficult.

Assuming y follow a *quasi-harmonic* model, meaning that it's modes are almost a sum of harmonics, how to find the relation between them ?
Considering (t, η), how to define a quantity $E(p, t, \eta)$ such that

- $E(p, t, \eta) >> 0$ if $\eta = p\phi'(t)$,
- $E(p, t, \eta)$ close to 0 otherwise.

Example of sliced STFT



Example of sliced STFT