Efficient methods in optimization

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example problems

1. Let

$$E_1 = \{x \mid (x - x_0)^T A(x - x_0) \leq 1\}, \quad E_2 = \{y \mid (y - y_0)^T B(y - y_0) \leq 1\} \tag{1}$$

be two given ellipsoids in $\mathbb{R}^n$, where $A, B \succ 0$.

Design a semi-definite program which checks the inclusion $E_1 \subset E_2$.

Hint: Homogenize and use the $\mathcal{S}$-lemma.

2. Consider again two ellipsoids $E_1, E_2 \subset \mathbb{R}^n$, given as in (1). We want to compute the distance between the ellipsoids.

- is this problem convex?
- can this problem be solved by an LP? an SOCP? an SDP?
- design a symmetric cone program which solves the problem.

3. Show that the Motzkin polynomial

$$x^4y^2 + x^2y^4 + z^6 - 3x^2y^2z^2$$

is not a sum of squares of other polynomials.

4. Consider the following network consisting of five intermediate tubes, one inlet tube, and one outlet tube. The inlet and outlet tubes have unlimited capacity, while the intermediate tubes have finite capacities $c_1, \ldots, c_5$ which are given in m^3/min units on Fig. 1. We want to determine the maximum possible flow from the inlet to the outlet tube through the network.

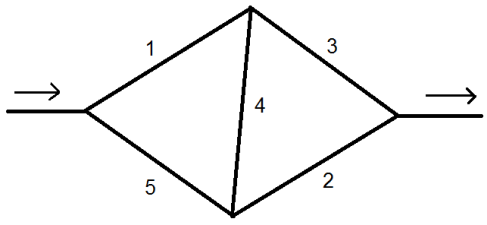

Figure 1: Network graph with flow capacities

Write this problem as a linear program.

Hint: There are balance constraints of equality type at each node and the constraints on the capacity.