Given a sequence of points $M_i = (x_i, y_i), 0 \leq i \leq n$, we want to construct a polynomial parametric curve of degree $n$ (because of the $n + 1$ constraints) which interpolates these data points, that is which goes through each point $(x_i, y_i)$ for some parameter $t_i$. The main question consists in the choice of the interpolation parameters $t_i$, which are also called the interpolation nodes or simply the nodes.

Precisely, we look for a $n$-degree polynomial parametric curve

$$m : \quad [a, b] \in \mathbb{R} \quad \rightarrow \quad \mathbb{R}^2 \quad t \quad \mapsto \quad m(t) = \left( \begin{array}{c} m_x(t) \\ m_y(t) \end{array} \right)$$

such that

$$m(t_i) = M_i \quad \Leftrightarrow \quad \begin{cases} m_x(t_i) = x_i \\ m_y(t_i) = y_i \end{cases}, \quad 0 \leq i \leq n$$

with a sequence of interpolation nodes $a \leq t_0 < t_1 < \cdots < t_n \leq b$, and where $m_x(t)$ and $m_y(t)$ are polynomials of degree $n$. As a result, we are reduced to solve two separate Lagrange interpolation problems.

We will consider three choices for the interpolation parameters. The first one is the uniform parametrization: parameters $t_i$ are evenly spaced in the parameters domain. The second one is the Chebyshev parametrization as introduced in the course on interpolation. The last one is the chordal parametrization: parameters are chosen in such a way that distances between successive parameters $t_i$ are proportional to the distances between associate successive data points $M_i$.

- Uniform parameterization: $t_i = a + i \frac{b-a}{n}$.

- Chebyshev parameterization: $\hat{t}_i = \frac{a+b}{2} + \frac{b-a}{2} \cos \left( (2i+1) \frac{\pi}{2n+2} \right)$.

- Chordal parameterization: $\tilde{t}_{i+1} - \tilde{t}_i = \frac{d_i}{\sum_{k=0}^{n-1} d_k}$ with $d_k = \text{dist}(M_k, M_{k+1})$ over $[0, 1]$.

Notice that the choice of the parameter domain $[a,b]$ does not affect the method, so that we will consider $[a,b] = [0,1]$. 
1) Récupérer le script Python “TP3parametricInterpolStudents.py” permettant d’acquérir à la souris un polygone dans une fenêtre graphique et de récupérer les coordonnées \((x_i, y_i)\) des sommets de ce polygone dans deux vecteurs \(x_i\) et \(y_i\):

ligne 83 : \(x_i, y_i = \text{PolygonAcquisition}('oc', 'c--')\)

Ce script fournit également les procédures permettant de réaliser l’interpolation de Lagrange dans la base de Newton à l’aide des différences divisées et de la méthode de Horner.

2) Compléter ce script afin de réaliser l’interpolation paramétrique des sommets du polygone acquis à la souris. Les 3 choix de points d’interpolation décrits ci-dessus seront implémentés.

2) Expérimenter ce script afin d’obtenir des figures semblables aux figures ci-dessous.

Il n’est pas demandé de compte rendu écrit pour ce TP — Néanmoins il sera testé en séance.