Practice for Math 4428 Final Exam

1. A bookseller wants to find how much he should spend on advertising for the new book he intends to sell, and how many copies he sould order. As a former student of M4428, he knows that he should order $u^*(k)$ copies defined by

$$\int_0^{u^*(k)} f_k(z)dz = \frac{p - c_0}{p + c_1}$$

where f_k is the density function of the random demand Z for an adversting level k, p is the price at which he sells a book, c_0 is the price at which he buys a book, and c_1 is the storage cost per book. He assumes that the demand is equally likely to fall between 0 and $b(k)$, with

$$b(k) = \begin{cases} 0 & \text{if } k \leq k_0 \\ \frac{1}{\alpha}\sqrt{k - k_0} & \text{if } k > k_0 \end{cases}$$

where k is the amount of money spent on advertising (unknown), k_0 given fixed setup costs, and α the cost to send one brochure of advertising.

   (a) Express $u^*(k)$ in terms of p, c_0, c_1, $b(k)$.

   (b) Find the corresponding expected reward

   $$J(u^*(k)) = (p + c_1)\int_0^{u^*(k)} z f_k(z)dz - k$$

   in terms of p, c_0, c_1, $b(k)$ and k.

   (c) Find a value of k which maximizes $J(u^*(k))$.

   (d) Application: The bookseller buys a book for \$10 per copy and sells them \$20 per book. Holding costs are \$1 per book. Concerning advertising, each sent brochure costs 10 cents, while fixed advertising costs are \$50. How much should he spend on advertising ? How many copies should he order ? What is its expected reward ?

2. Collecting weather data, it has been found that in Minneapolis, the day after a sunny day is 75% likely to be a sunny day, while the day after a rainy day is 50% likely to be a rainy day (we assume that a day is eiher sunny or rainy). Today is sunny.

   (a) What is the probability that it will rain in three days ?

   (b) At stationnary state (i.e. for large time) what is the probability of the occurence of a sunny day ?

   Hint: you can model it as a discrete-time two-floors elevator. For the second question you will have to diagonalize a two-by-two matrix A in the form $A = PDP^{-1}$, so that $A^n = PD^nP^{-1}$.