Math 4428 Midterm Exam

March 5, 2007

50 minutes closed book.

One double sided sheet of letter size paper allowed as document.

1. The cost of heating a room at temperature T_i is proportional to the yearly outgoing flux of heat across the walls enclosing the room. We assume that the outside temperature $T(t)$ increases from T_c at $t = 0$ to T_h at $t = 0.5$, and then decreases back to T_c for $t = 1$, the time unit being the year. Thus $t = 0.5$ corresponds to 6 months. The temperature is supposed to be a linear function of time from $t = 0$ to $t = 0.5$ and from $t = 0.5$ to $t = 1$. Let $\Delta T(t)$ be the temperature difference between T_i and $T(t)$ during the year. The heating is operating only when $\Delta T(t) > 0$. There is no cooling device.

   Let k and d denote the wall conductivity and thickness, respectively. The cost of the material per unit wall thickness is inversely proportional to k, while the cost of the wall is proportional to the material cost and the wall thickness.

   (i) (10 pts) We assume $T_c < T_i < T_h$. Can you justify this assumption ?

   (ii) (20 pts) Express the yearly outgoing heat flux in terms of T_i, T_c, T_h.

   (iii) (10 pts) Show that the yearly heating cost is proportional to

   $$\frac{k}{d}\frac{(T_i - T_c)^2}{T_h - T_c}$$

   (iv) (20 pts) Find a combination of conductivity and wall thickness that minimizes the total cost (one year of heating and initial construction).

   (v) (10 pts) Could you propose a more realistic, yet explicit, function $T(t)$?

2. (30 pts) For $\alpha > 0$ we consider the following differential system

   $$\begin{cases} \dfrac{dx}{dt} = x - \dfrac{1}{3}x^3 - y + \dfrac{1}{3}\alpha^3 \\ \dfrac{dy}{dt} = x - y \end{cases}$$

   Find and classify the equilibrium point of this system.