

Math 4428 Homework 7

due April 9, 2007

1. We study the growth of a population with a pure birth process as in 5.3.
 - (a) Is it realistic to consider a constant birth rate for a population growth model? Why?
 - (b) We make the simple assumption that $b_i = iB$, for $i \geq 1$, with B constant. Show from (5.46) that

$$r_{ij}(t) = \frac{(j-1)!}{(i-1)!(j-i)!} e^{-iBt} (1 - e^{-Bt})^{j-i}, \quad j \geq i \geq 1$$

Hint: Verify it first for $j = i$. Then assume it is true for j and show it is true for $j + 1$ (induction). The case $i = 0$ and $j = 0$ is not relevant here due to the form of the birth rate we took, and should not be considered.

2. Show (5.54) using the fact that for $|x| < 1$, the geometric series

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$$

converges absolutely and may be differentiated term by term. Choose suitable x .