

# Math 4428 Homework 1

due January 26, 2007

1. Problem 1.3. Additional: use the program 'implicitplot.m' on my homepage to obtain the graphs for the specified values of  $c$ .
2. Remember that the contagious disease model equation:

$$\frac{dR}{dt} = \gamma(n - S_0 e^{-\frac{R}{\mu}} - R) \quad (1)$$

was approximated by

$$\frac{dR}{dt} = \gamma \left( n - S_0 + \left( \frac{S_0}{\mu} - 1 \right) R - \frac{R^2 S_0}{2\mu^2} \right). \quad (2)$$

- (a) Show that the analytical solution of (2) is

$$R(t) = \frac{\mu^2}{S_0} \left( \left( \frac{S_0}{\mu} - 1 \right) + \delta \tanh\left(\frac{\gamma \delta t}{2} - \psi\right) \right)$$

where

$$\delta = \left( \frac{2S_0}{\mu^2} (n - S_0) + \left( \frac{S_0}{\mu} - 1 \right)^2 \right)^{\frac{1}{2}} \quad \psi = \operatorname{atanh}\left(\frac{1}{\delta} \left( \frac{S_0}{\mu} - 1 \right)\right)$$

- (b) Write a Matlab procedure to draw this exact solution, up to 2000 days. Use as data  $n = 100000$ ,  $S_0 = 99999$ ,  $a = 1$ ,  $\beta = 9 \times 10^{-6}$ ,  $\gamma = 0.89$ ,  $\mu = \frac{\gamma}{\beta}$ .
- (c) Modify it to draw on the same figure the numerical solution of (1), obtained using the `ode45` Matlab solver. Hint: define a Matlab function containing the left hand side of (1), and give it as argument to `ode45`.
- (d) Compare the two curves: could you explain the discrepancy? Hint: try to solve (2) numerically.
- (e) Try  $\beta = 10^{-5}$  and explain why the approximated model seems worst in that case.