Griffith Criterion

We see our ice floe as an elastic material, with rigidity tensor $A$. We represent it as a convex open set $\Omega \subset \mathbb{R}^2$. According to Griffith criterion [3, 2], the displacement $u$ and fracture $K$ are a minimum of the functional:

$$E_{\text{el}}(u,K) = E(u) + E_{\text{fr}}(K),$$

with the elastic energy:

$$E(u) = \int_{\Omega} A \varepsilon(u) : \varepsilon(u) \, dx,$$

and the fracture energy:

$$E_{\text{fr}}(K) = \int_{\partial K} 1 \, ds.$$

What boundary conditions should we apply when a collision happens? We know what happens when an object collides with a spring network. We intend to show that a spring network indeed represents an elastic material.

We need an isotropic lattice. We draw from a Poisson point process, and we build on top of it the associated Voronoi tessellation, and its dual, the Delaunay triangulation.

An isotropic network of springs

We denote $\Phi_0$ as the Poisson point process of intensity $\lambda$. As in [4], we compute the law of the typical triangle:

$$P_0(A) = \int A \lambda^{d-1} \exp(-\lambda\cdot\Theta_0) \, d\Theta_0 \cdot dA,$$

where $\Theta_0$ is the Poisson point process of intensity $\lambda^0$. The typical triangle $A$ is:

$$E_{\text{el}}(u,K) \to E$$

where the rescaled energies $E_{\text{el}}(u,K) \to E$ are defined on the spaces $W(\tau_0)$ by:

$$E_{\text{el}}(u) = \varepsilon^2 E(u),$$

and the elastic energy $E: H^1(\Omega) \to \mathbb{R}$ is:

$$E(u) = \frac{1}{2} \int A \varepsilon(u) : \varepsilon(u) \, dx,$$

with $C$ the stiffness tensor with Hooke's coefficient: $(\tau = \frac{\lambda}{\lambda^0}).$

Phase-field fracturation

A result of Ambrosio and Torelli [1] would allow us to use the approximation functionals:

$$E_\varepsilon(u,v) = \frac{1}{2} \int_{\Omega} \varepsilon(u) : \varepsilon(u) \, dx + \int_{\Omega} |\nabla u|^2 \, dx + \frac{1}{2} \int_{\Omega} (1 - v)^2 \, dx.$$

Problem: we would need a tight mesh!

Efficient fracturation method

We assume that the fracture $K$ is a line. We have to minimize the following:

$$\int_{\partial K} A \varepsilon(u) : \varepsilon(u) \, ds + H^1(K),$$

where $u$ belongs to the variational space $V_K = H^1(\Omega \setminus K).$

To compare the different functional spaces, we prove Mosco convergence of the $V_K$.

Theorem: if the lines $K_n$ converge to $K$, the functional spaces $V_{K_n}$ converge in the Mosco sense to $V_K$, that is:
1. for all sequences $(u_n \in V_{K_n})$ with $u_n \to u$ in $L^2(\Omega)^2$, then $u \in V_K$,
2. for all $u \in V_{K_n}$, there exist a sequence $(u_n \in V_{K_n})$ such that $u_n \to u$ in $L^2(\Omega)^2$.

The existence of a minimum follows.

Finite element method

On the finite element space $W_0^p(\Omega \setminus K)$, we use the following scalar product:

$$(u,v) = \int_K u v \, dx.$$

In doing so, we allow fracture location to be independent of mesh precision.

Bibliography