Vortex penalization method for solid-porous-fluid media with application to passive flow control

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ABSTRACT
A coupling of vortex methods with penalization methods is proposed in this work in order to accurately and easily handle solid-fluid-porous media. This immersed boundary approach indeed maintains the efficiency and the robustness of vortex methods and allows to model the three different media without prescribing any boundary condition. In this paper, we propose an application of this immersed boundary method to passive flow control around a semi-circular cylinder, realized adding a porous sheath on the obstacle surface in order to smooth the flow dynamics.

The model
In this work, flow simulations are based on particle methods. The fluid particles which are displaced by convection and diffusion are characterized by their position and their vorticity. The vorticity transport is expressed by the Vorticity Transport Equation, obtained taking the curl of the incompressible Navier-Stokes equations and given in 2D by:

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \frac{1}{\text{Re}} \Delta \omega - \nabla \times [\lambda S \mathbf{u}_b - \mathbf{u}],$$

where $\omega$, $\mathbf{u}$ and Re respectively denote the vorticity, the velocity and the Reynolds number. The Poisson equation $\nabla^2 \mathbf{u} = - \nabla \times \omega$ obtained from continuity equation, enables to recover velocity field once the vorticity field is known. The previous equations are approximated using a remeshed Vortex method [1] based on a two-fractional step method. It relies on approximating separately the diffusion and convection terms at each time step. The convective part is solved using a ”Vortex-In-Cell (VIC)” method (see e.g. [1]) with a semi-Lagrangian resolution: the fluid particles are displaced with the convective velocity and then remeshed on the original grid in order to avoid Lagrangian distortion. Finally, for computational efficiency and accuracy, diffusion and Poisson equation are solved on the grid using Fast Fourier Transforms (FFT).

This work is devoted to the study of flows in solid-porous-fluid media and the method used to handle such problems is the vortex penalization technique. This immersed boundary method appears as a very good approach since it involves a unique equation for the whole domain. The latter is the non-dimensional Brinkman-Vorticity Transport Equation obtained from Eq. (1) by adding a penalization term and expressed as:

$$\frac{\partial \omega}{\partial t} + (\mathbf{u}, \nabla) \omega = \frac{1}{\text{Re}} \Delta \omega + \nabla \times [\lambda S \mathbf{u}_b - \mathbf{u}],$$

where $S$ denotes the characteristic function that yields 0 in the fluid and 1 in the solid, $\mathbf{u}_b$ indicates the rigid body velocity which is zero in all this work since the body is fixed and $\lambda = \mu \Phi l_{ef} / \rho l_{ref}$ is the non-dimensional penalization parameter, with $k$ the intrinsic permeability, $\mu$ the viscosity, $\Phi$ the porosity of the porous material, $l_{ef}$ the height of the obstacle, $\rho$ the density and $l_{ref}$ the reference velocity. Since $l_{ef}, \rho, l_{ref} = 1$ and since the porosity $\Phi$ is close to 1 as imposed by Brinkman equations [2], $\lambda$ essentially depends, in the inverse proportion, on the intrinsic permeability $k$ of the medium. Varying the $\lambda$ value thus directly defines the different media according to the following equation obtained using implicit Euler scheme for the penalization velocity discretization:

$$\mathbf{u}_{n+1} = \mathbf{u}_{n} \frac{1}{1 + \lambda \Delta t S}.$$

Indeed, in the fluid, the intrinsic permeability coefficient $k$ goes to infinity, thus the fluid can be considered numerically as a porous media with a very high permeability. We set $\lambda = 0$ in this region. As a consequence, according to Eq. (3), the velocity in the fluid is not penalized ($\mathbf{u}_{n+1} = \mathbf{u}_{n}$) and since $\lambda = 0$, the penalization term vanishes in Eq. (2), and we naturally recover the Vorticity Transport Equation (Eq. 1). On the contrary, the solid has a permeability coefficient $k$ which goes to zero, it can be consequently modeled setting the penalization parameter $\lambda$ to a very high value. In this study $\lambda$ equals $10^3$ in the solid, which vanishes the flow velocity in this region according to Eq. (3) ($\mathbf{u}_{n+1} \rightarrow 0$). It was proved in [3] that solving Eq. (2) with such a value of $\lambda$ was equivalent to solve Darcy’s law in the solid. Furthermore, setting the $\lambda$ parameter to an intermediate value, reasonably chosen between these two extreme values ($\lambda = 0$ and $\lambda = 10^3$), would model a porous medium in which the flow has a Darcy velocity.
As a conclusion, the variation of $\lambda$ corresponds to the variation of $\kappa$ that specifies the intrinsic porous material permeability. The accuracy and efficiency of the penalization method come from its capability to take into account these variations of $\lambda$ and to capture the induced steep velocity variations at the different interfaces with a minimum number of discretization points.

**Application to passive flow control**

Modeling the physics of three different regions enables one to deal with engineering problems involving porous media. In the following, this approach is validated for a simple but significant passive flow control problem.

Here, the solid-porous-fluid configuration is applied to cover a semi-circular cylinder geometry with a porous coating. The latter is settled on the obstacle external surface in order to modify the vorticity generation of the boundary layer and the vortex shedding. In fact, the presence of a porous medium at the solid-fluid interface imposes a kind of mixed boundary condition intermediate between the no-slip and the slip one on the solid boundary. As a result, the shear forces are decreased and the flow dynamics is smoothed [4, 5]. The semi-circular cylinder can be considered as a simplified section of an outside rear-view mirror of a car or a motor cycle. The mirrors, due to their spanwise position, indeed generate a non-negligible wake which interferes with the flow past vehicle sides. This accounts for a good motivation to perform flow control past such obstacles.

The subsequent flow control simulations are performed at transitional (Re = 550) and highly transitional regime (Re = 3000). The semi-circular cylinder has a total dimensionless diameter of $d = 1$ including a porous layer of thickness $\tau = 10% d = 0.1$ and whose back wall is centered at $(x, y) = (0, 0)$ in the computational domain $D = [-4, 8] \times [-5, 5]$ (Fig. 1 (left)). The whole computational domain is meshed by an uniform Cartesian orthogonal grid with a space step $h$ equal to 0.005 at Re = 550 and $h = 0.0025$ at Re = 3000. As we use FFT-based evaluations to solve diffusion and Poisson equation, periodic boundary conditions are considered on the box walls $\Gamma_D$. To highlight the influence of the added porous layer permeability on the flow control behaviour and the efficiency of such a passive control, a parametric study is firstly performed considering four values of the porous permeability inside the layer, namely $\lambda = 1$ (high permeability), $10. 10^2$, $10^3$ (low permeability) and comparing the results to the solid ($\lambda = 10^8$) and fluid ($\lambda = 0$) cases. For this parametric study, the added layer is homogeneous (Fig. 1 (right)). An heterogeneous configuration will be addressed secondly.

Figure 1: (left) Computational domain. (right) Geometrical cases corresponding to uncontrolled/controlled devices

In order to analyze the effects of our control approach we compare global flow quantities like the drag force ($F_D$), computed according to the momentum equation [6] and the enstrophy ($Z$), expressed as $Z = \int_D |\omega|^2 \, dx$, allowing to measure the dissipation effects in the flow as well as the delay of transition to turbulence. As can be seen in Fig. 2, which represents dimensionless time history of global flow quantities at Re=550, setting $\lambda = 1$ inside the layer clearly appears as the best solution in terms of flow regularization. Indeed, the mean value of drag force (Fig. 2 (a)) reaches for $\lambda = 1$ an optimum value close to the one of the fluid case, showing a drag reduction of about 30% compared to uncontrolled case. Results of enstrophy evolution (Fig. 2 (b)) show that the result obtained with $\lambda = 1$ is even better than the one of fluid case and represents an improvement of nearly 40% compared to uncontrolled case. At Re = 3000 the best solution for global flow regularization is also achieved setting the $\lambda$ parameter to 1 inside the porous coating. This configuration leads to a drag and enstrophy reduction of respectively 21% and 44% compared to the uncontrolled case (Fig. 2 (c) and (d)).

The position of the permeable zone also has a great importance in passive control effects. To illustrate this statement, we perform a comparison between the homogeneous layer configuration and the heterogeneous one where the porous zone is only located on top and bottom of the semi-circular cylinder (Fig. 1 (right)). According to the previous parametric study, the value of $\lambda$ in the permeable zone is set to 1 for both configurations. Surprisingly, at Re=550 as well as Re=3000, the heterogeneous device shows benefits which are better than the one obtained with the homogeneous one. We note that the only difference with the uncontrolled case is the presence of high permeable poles in the layer. As can be seen in Fig. 3 (left), the latter allow an eddy detachment from the wall and a decrease of the transverse dimension of the wake. Moreover the near wake structures are smaller and the back recirculation zone is sharply reduced, implying a drastic increase of downstream pressure (Fig. 3 (right)) and thus a significant drag force reduction, which is about 45% in both regimes. Since it represents a good compromise between manufacturing constraints and control efficiency, these results make the porous-poles configuration a suitable device for passive flow control.

**REFERENCES**

Figure 2: Effects of various layer permeabilities on drag force ($F_D$) and enstrophy ($Z$) related to flow past a semi-circular cylinder at $Re=550$ ((a) and (b)) and $Re=3000$ ((c) and (d)).

Figure 3: (left) Zoom of the mean vorticity fields and isolines at $Re=550$ and $Re=3000$. (right) Mean pressure profiles at the rear end of the body at $Re=550$ (top) and $Re=3000$ (bottom).