MINI-COURSE: THE LEVEL SET METHOD IN CONNECTION WITH MESHING

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Since its introduction in the context of curvature-driven interface motion [2], the level set method has been one choice strategy for describing the time evolution of a domain.

Crucially, this paradigm hinges on a change of perspectives: a domain $\Omega \subset \mathbb{R}^d$ is seen as the negative subdomain of an auxiliary "level set" function $\phi : D \to \mathbb{R}$, defined on a fixed computational domain $D \subset \mathbb{R}^d$:

$$\forall x \in D, \quad \begin{cases} \phi(x) < 0 & \text{if } \; x \in \Omega, \\ \phi(x) = 0 & \text{if } \; x \in \partial\Omega, \\ \phi(x) > 0 & \text{otherwise.} \end{cases}$$

The time evolution of a domain $\Omega(t) \subset D$ along a velocity field $V(t,) : D \to \mathbb{R}^d$ then translates in terms of a partial differential equation for an associated level set function $\phi(t, \cdot) : D \to \mathbb{R}$, which can be conveniently solved on a fixed mesh of D [1, 3].

This mini-course is devoted to the recent applications of the level set method has found multiple applications in connection with mesh processing.

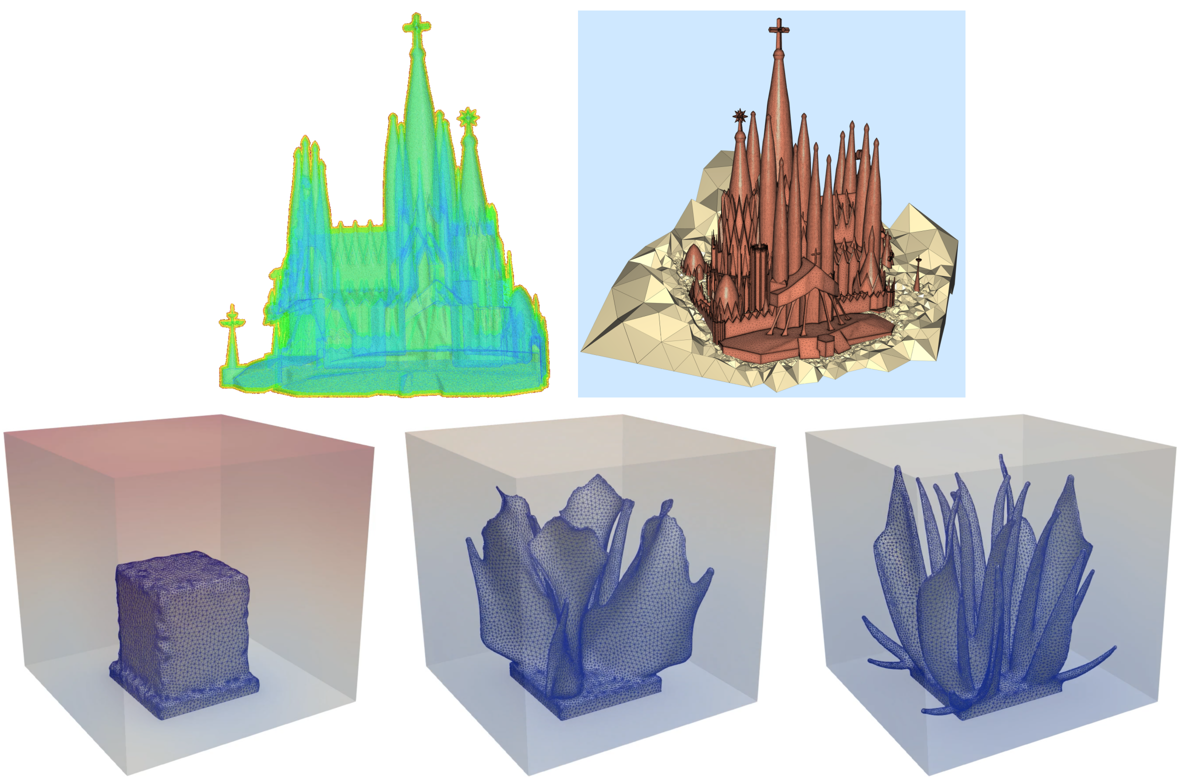

Figure 1. *(Upper row) Construction of a computational mesh (on the right) from the datum of a distance function (a few isosurfaces of which are represented on the left); (Lower row) Evolution of the mesh of a domain through a shape and topology optimization process.*

In a first part, we shall present the basic ingredients of the level set method, and notably the mathematics of implicitly-defined domains and surfaces, the numerical construction of level set functions as signed distance functions, and the tracking of the motion of a domain in the framework of the level set method.

In a second time, we discuss several applications of this method which are more specifically connected with meshing, in particular,

- The reconstruction of a valid computational mesh from an input messy, invalid surface triangulation;
- The tracking of the evolution of a mesh of a domain undergoing arbitrarily large motions, including changes in its topology.

References

[1] S. Osher and R. Fedkiw, *Level set methods and dynamic implicit surfaces*, vol. 153, Springer Science & Business Media, 2006.

[2] S. Osher and J. A. Sethian, *Fronts propagating with curvature-dependent speed: algorithms based on hamilton-jacobi formulations*, Journal of computational physics, 79 (1988), pp. 12–49.

[3] J. A. Sethian, *Level set methods and fast marching methods: evolving interfaces in computational geometry, fluid mechanics, computer vision, and materials science*, vol. 3, Cambridge university press, 1999.