Hybrid approach for linear time-varying systems: Hovering for impulsive spacecraft rendezvous

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Orbital rendezvous definition

Orbital Rendezvous consists of:

- Keplerian relative motion;
- Relative navigation;
- Passive Target;
- Fixed-time;
- Usage of ergols thrusters:
  - the control is modeled by impulsive signals,
  - Instantaneous velocity change,
  - Sequence of coasting arc limited by thruster impulses;

Rendez-vous problem

- Steering the chaser spacecraft from a state $A$ to state $B$;
- Assuming some operating constraints (Actuators bounds, safety constraints ...);
Hovering problem for rendez-vous

Needs of stationkeeping abilities for different purposes

- waiting for orders from the ground station to proceed the mission
- observation mission of passive spacecraft
Hovering problem for rendez-vous

Needs of stationkeeping abilities for different purposes

- waiting for orders from the ground station to proceed the mission
- observation mission of passive spacecraft

Literature Background:

- Computing maneuver making the spacecraft bouncing on the frontier of the hovering subset.
Hovering problem for rendez-vous

Needs of stationkeeping abilities for different purposes

- waiting for orders from the ground station to proceed the mission
- observation mission of passive spacecraft

Our approaches:

- Taking advantages of natural periodic orbits to decrease the consumption.
Periodic orbits approaches for hovering

- **Hybrid control approach**
  - Stabilisation of a particular admissible periodic orbit

- **MPC control approach**
  - Stabilisation toward the set of admissible periodic orbit
Relative motion dynamics I

Space mechanics assumptions

- Keplerian relative motion: perfectly spheric Earth, no aerodynamic drag, no influence from sun and moon
- Passive Target evolving on elliptical orbits;

Non linear dynamics are given by:

\[
\begin{align*}
\ddot{x} &= 2 \dot{\nu} \dot{z} + \dot{\nu} z + \dot{\nu}^2 x - \frac{\mu x}{\sqrt{(x^2 + y^2 + (R - z)^2)^3}}, \\
\ddot{y} &= -\frac{\mu y}{\sqrt{(x^2 + y^2 + (R - z)^2)^3}}, \\
\ddot{z} &= -2 \dot{\nu} \dot{x} - \dot{\nu} x + \dot{\nu}^2 z - \frac{\mu (R - z)}{\sqrt{(x^2 + y^2 + (R - z)^2)^3}} + \frac{\mu}{R^3}.
\end{align*}
\]

State space vector

\[ X(t) = [x, \ y, \ z, \ \dot{x}, \ \dot{y}, \ \dot{z}] \]
Relative motion dynamics II

Linearization assumption:

- Relative navigation: distance between satellite is small with respect to the radius of the target spacecraft.

Linearized equation for the relative motion can be obtained:

Tschauner-Hempel equations

\[
\begin{align*}
\ddot{x} &= \dot{\nu} \ddot{z} + \ddot{\nu} z + \dot{\nu}^2 x - \frac{\mu}{R^3} x \\
\dot{\nu} &= -\frac{\mu}{R^3} x \\
\ddot{y} &= -2\dot{\nu} \ddot{x} - \dot{\nu} x + \dot{\nu}^2 z + 2 \frac{\mu}{R^3} z
\end{align*}
\]

Substitution of the time variable \( t \) by the target true anomaly \( \nu \) and the change of variable

\[
\tilde{X}(\nu) = \begin{bmatrix} (1 + e \cos \nu)I_3 & 0_3 \\ -e \sin \nu I_3 & \frac{(1 + e \cos \nu)}{\dot{\nu}} I_3 \end{bmatrix} X(t) \quad \text{with} \quad \dot{\nu} = \frac{2\pi}{T} \frac{(1 + e \cos \nu)^2}{(1 - e^2)^{3/2}}
\]

Simplified Tschauner-Hempel eqs.

\[
\begin{align*}
\dddot{x} &= 2\dddot{z}' \\
\dddot{y} &= -\dddot{y} \\
\dddot{z} &= -2\dddot{x}' + \frac{3}{1 + e \cos \nu} \dddot{z}
\end{align*}
\]
Hybrid Modeling for the rendezvous problem I

Time-varying model

\[
\begin{align*}
\dot{\tilde{X}} &= A(\nu)\tilde{X} \\
\tilde{X}^+ &= \tilde{X} + \begin{bmatrix} 0_{3\times3} \end{bmatrix} u
\end{align*}
\]

- in free motion
- at impulse execution

\[
A(\nu) = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 2 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{1 + e \cos \nu} & -2 & 0 & 1
\end{bmatrix}
\]
Hybrid Modeling for the rendezvous problem II

Floquet-Lyapunov transformation

\[ \xi = G(\nu)\tilde{X} := S(\nu) C(\nu) \tilde{X}, \]

- \( C(\nu) := \begin{bmatrix} 0 & c_\nu & 0 & 0 & -s_\nu & 0 \\ 0 & s_\nu & 0 & 0 & c_\nu & 0 \\ 1 & 0 & -3es_\nu(1+\rho) & es_\nu(1+\rho) & 0 & \rho^2e^{-e\nu_3} \\ e & 0 & -3s_\nu & s_\nu(1+\rho) & 0 & c_\nu \rho \\ 0 & 0 & 3(c_\nu+e) & c_\nu(1+\rho)+e & 0 & s_\nu \rho \\ 0 & 0 & -3(3c_\nu+e^2+2) & 3\rho^2 & 0 & -3es_\nu \rho \end{bmatrix} \]

- \( S(\nu) := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \)

\[ \sigma(\nu) = (\nu - \nu_f) - n(t - t_f) \]
Hybrid Modeling for the rendezvous problem III

New coordinates $\xi$ dynamics

LTI flow dynamics:

- $\xi' = A_{\xi} \xi$ in free motion
- $\xi' = A_{\xi} \xi + B_{\xi}(\nu)u$ at impulse execution

$$A_{\xi} = (1-e^2)^{-3/2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{\xi}(\nu) = \frac{1}{k^2 \rho^2 (1-e^2)} \begin{bmatrix} 0 & -(1-e^2) \rho s_{\nu} & 0 \\ 0 & (1-e^2) \rho c_{\nu} & 0 \\ -(1+\rho) \rho s_{\nu} - \frac{3\sigma \rho^3}{(1-e^2)^{3/2}} & 0 & \frac{3\sigma \rho^2 s_{\nu}}{(1-e^2)^{3/2}} - \rho^3 + \rho^2 - 2\rho \\ (1-e^2)(1+\rho) \rho s_{\nu} & 0 & (1-e^2) \rho^2 c_{\nu} \\ (1+\rho) \rho c_{\nu} + \epsilon \rho & 0 & -\rho^2 s_{\nu} \\ -3\rho^3 & 0 & 3\epsilon \rho^2 s_{\nu} \end{bmatrix}$$

Equilibrium and periodicity

- $\xi_6 = 0$ implies that $\xi$ is constant
- Constant $\xi \iff$ Periodic orbits
Hybrid Control laws

New hybrid model

\[(\xi, \nu, \tau) \in C \quad | \quad (\xi, \nu, \tau) \in D_\nu \quad | \quad (\xi, \nu, \tau) \in D_u\]

\[
\begin{align*}
\xi' &= A\xi, \\
\nu' &= 1, \\
\tau' &= -1,
\end{align*} \quad \begin{align*}
\xi^+ &= \xi, \\
\nu^+ &= 0, \\
\tau^+ &= \tau,
\end{align*} \quad \begin{align*}
\xi^+ &= \xi + B\xi(\nu)\gamma_u(\xi, \nu), \\
\nu^+ &= \nu, \\
\tau^+ &= \gamma_\tau(\xi, \nu),
\end{align*}
\]

where

- \(C := (\mathbb{R}^6 \times [0, 2\pi] \times [0, 2\pi]) \setminus D_\nu \cap D_u\)
- \(D_\nu := \mathbb{R}^6 \times \{2\pi\} \times [0, 2\pi],\)
- \(D_u := \mathbb{R}^6 \times [0, 2\pi] \times \{0\},\)

The degree of freedom this hybrid approach are

Control law \(\gamma_u(\cdot);\)

Trigger law \(\gamma_\tau(\cdot).\)

1. Periodic bi-impulsive control:
   - \(\gamma_u(\cdot)\) based on transition inversion;
   - \(\gamma_\tau(\cdot)\) fixed \textit{a priori}.

2. Periodic norm-minimizing control:
   - \(\gamma_u(\cdot)\) minimizes the 2-norm of impulse;
   - \(\gamma_\tau(\cdot)\) fixed \textit{a priori}.

3. Non periodic bi-impulsive control:
   - \(\gamma_u(\cdot)\) based on transition inversion;
   - \(\gamma_\tau(\cdot)\) optimized over one period.
Periodic bi-impulsive control law

Control law $\gamma_u$

- $\gamma_u$ computes $u(\nu_i)$ and $u(\nu_i + \bar{\nu})$ is such that:

$$\xi_{ref} - \xi^+(\nu_i + \bar{\nu}) = \tilde{\xi}^+(\nu_i + \bar{\nu}) = 0$$

- $\gamma_u$ is given by

$$\gamma_u(\xi(\nu_i), \bar{\nu}) = [-I \ 0]M(\nu, \bar{\nu})^{-1}\tilde{\xi}. $$

where

$$M(\nu, \bar{\nu}) := \begin{bmatrix} \hat{B}(\nu) & \Phi(-\bar{\nu})\hat{B}(\nu + \bar{\nu}) \end{bmatrix} \quad \Phi(\mu) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \mu(1-e^2)^{-3/2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- Matrix $M$ is inversible $\iff \bar{\nu} \neq k\pi$, $k \in \mathbb{Z}$

Trigger law $\gamma_\tau$

Periodic trig of the control law: $\gamma_\tau = \bar{\nu}$

Theorem

The reference periodic orbit $\xi_{ref}$ is stable and attractive
Periodic bi-impulsive norm-minimizing control law

Control law $\gamma_u$

- $\gamma_u$ computes $u(\nu_i)$ such that:
  \[ u^* = \arg\min_u |\tilde{\xi}^+|^2, \quad \text{such that: } \tilde{\xi}^+ = \tilde{\xi} + \hat{B}(\nu)u, \quad \tilde{\xi}_6^+ = 0 \]

- $\gamma_u$ is given by
  \[
  \gamma_u(\tilde{\xi}, \nu) = u_6 - \hat{B}_6^\perp(\nu)(\hat{B}(\nu)\hat{B}_6^\perp(\nu))^{-L}(\tilde{\xi} + \hat{B}(\nu)u_6) \quad \text{with } u_6 = -\frac{\hat{b}_6(\nu)}{|\hat{b}_6(\nu)|^2}\tilde{\xi}_6,
  \]

where

\[
\hat{b}_6(\nu) = \begin{bmatrix}
\frac{3\rho^3}{e^2 - 1} \\
\frac{3\rho^2 e \sin(\nu)}{1 - e^2} \\
0
\end{bmatrix}, \quad \hat{B}_6^\perp(\nu) = \begin{bmatrix}
e \sin(\nu) & 0 \\
0 & 1 \\
\rho(\nu) & 0
\end{bmatrix}, \quad \hat{b}_6(\nu)\hat{B}_6^\perp(\nu) = 0
\]

Trigger law $\gamma_\tau$

Periodic trig of the control law: $\gamma_\tau = \bar{\nu}$

Theorem

The reference periodic orbit $\xi_{ref}$ is stable (attractivity non guaranteed)
Aperiodic bi-impulsive control law

Control law $\gamma_u$

- $\gamma_u$ computes $u(\nu_i)$ and $u(\nu_i + \bar{\nu})$ is such that:

$$\xi_{ref} - \xi^+(\nu_i + \bar{\nu}) = \tilde{\xi}(\nu_i + \bar{\nu}) = 0$$

- $\gamma_u$ is given by

$$\gamma_u(\xi(\nu_i), \bar{\nu}) = [-I \ 0]M(\nu, \bar{\nu})^{-1}\tilde{\xi}.$$  

with $M(\nu, \bar{\nu}) := [\hat{B}(\nu) \ \Phi(-\bar{\nu})\hat{B}(\nu + \bar{\nu})]$  

$\bar{\nu} \neq k\pi, \ k \in \mathbb{Z}$

Trigger law $\gamma_\tau$

- Search for the best moment to trig:

$$\gamma_\tau(\tilde{\xi}, \nu) = \text{argmin}_{\bar{\nu}\in[0,2\pi]} \left|M(\nu, \bar{\nu})^{-1}\tilde{\xi}\right|_1$$

Theorem

The reference periodic orbit $\xi_{ref}$ is stable and attractive.
Numerical studies first example

\[ \begin{align*}
\text{mission 1} \\
a = 7011 \text{km}, \ e = 0.1, \ \bullet \ \xi^{ref} = [5.2, 17.7, 98.0, 22.5, -17.6, 0], \\
\bullet \ \xi_01 = [500, 400, 10, 0, 0, 0], \ \bullet \ \xi_02 = [-200, 100, 200, 0, 0, 0], \ \bullet \ \xi_03 = [100, -350, -20, 0, 0, 0], \\
\text{with} \ \nu_01 = 10^\circ, \ \nu_02 = 150^\circ, \ \nu_03 = 60^\circ
\end{align*} \]
### Numerical studies first example

**mission 1**

\[a = 7011\text{km}, \, e = 0.1, \, \mathbf{\xi}_{\text{ref}} = [5.2, 17.7, 98.0, 22.5, -17.6, 0],\]

\[\mathbf{\xi}_0 = [500, 400, 10, 0, 0, 0], \quad \mathbf{\xi}_2 = [-200, 100, 200, 0, 0, 0], \quad \mathbf{\xi}_3 = [100, -350, -20, 0, 0, 0],\]

**with** \(\nu_0 = 10^\circ, \, \nu_2 = 150^\circ, \, \nu_3 = 60^\circ\)

---

**Periodic Norm-minimizing Control, \(\vec{\nu} = \pi/2\)**

- \(\max (|\xi_{1..3}|_B) \quad (\text{m})\):
  - \(\mathbf{\xi}_0 = 1.1192\)
  - \(\mathbf{\xi}_2 = 1.0188\)
  - \(\mathbf{\xi}_3 = 0.5311\)

- \(\max (|\xi_{1..3}|_B) \quad (\text{deg})\):
  - \(\mathbf{\xi}_0 = 1547.5395\)
  - \(\mathbf{\xi}_2 = 1300.7956\)
  - \(\mathbf{\xi}_3 = 87.1149\)

---

**Periodic Bi-impulsive Control, \(\vec{\nu} = \pi/2\)**

- \(\max (|\xi_{1..3}|_B) \quad (\text{m})\):
  - \(\mathbf{\xi}_0 = 1.5206 (+35.86\%)\)
  - \(\mathbf{\xi}_2 = 0.6550 (-35.71\%)\)
  - \(\mathbf{\xi}_3 = 0.5184 (-2.39\%)\)

- \(\max (|\xi_{1..3}|_B) \quad (\text{deg})\):
  - \(\mathbf{\xi}_0 = 88.7732 (-94.26\%)\)
  - \(\mathbf{\xi}_2 = 80.4853 (-93.81\%)\)
  - \(\mathbf{\xi}_3 = 87.1149 (+0.00\%)\)

---

**Aperiodic Bi-impulsive Control, \(\vec{\nu} = \nu_{\text{opt}}\)**

- \(\max (|\xi_{1..3}|_B) \quad (\text{m})\):
  - \(\mathbf{\xi}_0 = 0.6552 (-41.46\%)\)
  - \(\mathbf{\xi}_2 = 0.6098 (-40.15\%)\)
  - \(\mathbf{\xi}_3 = 0.4877 (-8.17\%)\)

- \(\max (|\xi_{1..3}|_B) \quad (\text{deg})\):
  - \(\mathbf{\xi}_0 = 265.8172 (-82.82\%)\)
  - \(\mathbf{\xi}_2 = 246.2420 (-81.07\%)\)
  - \(\mathbf{\xi}_3 = 81.7291 (-6.18\%)\)
Numerical studies first example

**mission 1**

\[ a = 7011 \text{km}, \ e = 0.1, \ \xi_{r e f} = [5.2, 17.7, 98.0, 22.5, -17.6, 0], \]

\[ \xi_01 = [500, 400, 10, 0, 0, 0], \ \xi_02 = [-200, 100, 200, 0, 0, 0], \ \xi_03 = [100, -350, -20, 0, 0, 0], \]

with \( \nu_01 = 10^\circ, \ \nu_02 = 150^\circ, \ \nu_03 = 60^\circ \)
Periodic Norm-minimizing Control, $\bar{\nu} = \pi/2$

Periodic Bi-impulsive Control, $\bar{\nu} = \pi/2$

Aperiodic Bi-impulsive Control, $\bar{\nu} = \nu_{\text{opt}}$

Numerical studies first example

mission 1

$a = 7011 \text{km}, e = 0.1, \bullet \xi^{\text{ref}}_f = [5.2, 17.7, 98.0, 22.5, -17.6, 0], \bullet \xi_{01} = [500, 400, 10, 0, 0, 0], \bullet \xi_{02} = [-200, 100, 200, 0, 0, 0], \bullet \xi_{03} = [100, -350, -20, 0, 0, 0],$

with $\nu_{01} = 10^\circ, \nu_{02} = 150^\circ, \nu_{03} = 60^\circ$
Numerical studies first example

**mission 1**

\[ a = 7011 \text{km}, \ e = 0.1, \ \mathbf{\xi}^{ref} = [5.2, 17.7, 98.0, 22.5, -17.6, 0], \]

\[ \mathbf{\xi}_0 = [500, 400, 10, 0, 0, 0], \ \mathbf{\xi}_0 = [-200, 100, 200, 0, 0, 0], \ \mathbf{\xi}_3 = [100, -350, -20, 0, 0, 0], \]

with \( \nu_0 = 10^\circ, \ \nu_0 = 150^\circ, \ \nu_3 = 60^\circ \)

Periodic Norm-minimizing Control, \( \bar{\nu} = \pi/2 \)

Periodic Bi-impulsive Control, \( \bar{\nu} = \pi/2 \)

Aperiodic Bi-impulsive Control, \( \nu_{opt} \)
Numerical studies first example

mission 1

\[ a = 7011\, \text{km}, \, e = 0.1, \, \xi^{ref} = [5.2, 17.7, 98.0, 22.5, -17.6, 0], \]

\[ \xi_{01} = [500, 400, 10, 0, 0, 0], \, \xi_{02} = [-200, 100, 200, 0, 0, 0], \, \xi_{03} = [100, -350, -20, 0, 0, 0], \]

with \( \nu_{01} = 10^\circ, \, \nu_{02} = 150^\circ, \, \nu_{03} = 60^\circ \)
mission 2

\( a = 7011 \text{km}, e = 0.4, \bullet \xi^{ref} = [7.7, 17.7, 87.8, 33.0, -15.8, 0], \)

\( \bullet \xi_01 = [500, 400, 10, 0, 0, 0], \bullet \xi_02 = [-200, 100, 200, 0, 0, 0], \bullet \xi_03 = [100, -350, -20, 0, 0, 0], \)

with \( \nu_01 = 10^\circ, \nu_02 = 150^\circ, \nu_03 = 60^\circ \)
**Numerical studies: second example**

\[ a = 7011 \text{ km}, \ e = 0.4, \ \xi^{ref} = [7.7, 17.7, 87.8, 33.0, -15.8, 0], \]
\[ \xi_{01} = [500, 400, 10, 0, 0, 0], \ \xi_{02} = [-200, 100, 200, 0, 0, 0], \ \xi_{03} = [100, -350, -20, 0, 0, 0], \]

with \[ \nu_{01} = 10^\circ, \ \nu_{02} = 150^\circ, \ \nu_{03} = 60^\circ. \]
Conclusions

Submitted paper

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“A hybrid control framework for impulsive control of satellite rendezvous”.

Achievements

- Use of Floquet-Lyapunov to obtain LTI flow;
- Hybrid description of the hovering problem;
- Stability and convergence (in most of cases) is guaranteed.

Future works

- Accounting for thrusters saturations;
- Evaluate the robustness through non-linear simulations;
- Address the robust stability properties of the presented control law.
Model Predictive Control for hovering station zone

Constrained periodic orbits SDP description

\[
S^P(H, V) = \left\{ D(w_0) \in \mathbb{R}^6 \ \bigg| \ \begin{array}{l}
d_3(t) = 0 \\
\exists Y_i \succeq 0 \text{ t.q. } \gamma_i^P = \Lambda^*(Y_i)
\end{array} \right\}
\]