A Lyapunov redesign of coordination algorithms for systems subject to resource constraints

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Problem statement

**Systems in network**
- Inteconnection graph $\mathcal{G} = (\mathcal{I}, \mathcal{E})$
- Node dynamics ($i \in \mathcal{I}$)

\[
\begin{align*}
\dot{p}_i &= v_i \\
\dot{v}_i &= -v_i + u_i
\end{align*}
\]

**Control objective**
Rendez-vous, i.e. $p_i(t) - p_j(t) \to 0$ as $t \to \infty$
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- Continuous measurements
- Continuous control updates
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To limit control updates → Event-triggered control

- to slow down actuator wear
- to reduce actuators energy consumption
**Problem statement**

To limit control updates $\rightarrow$ Event-triggered control

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![Diagram](image)
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To limit control updates and communication → Self-triggered control

+ to limit the network usage
+ to reduce sensors batteries consumption
- more control updates than event-triggered control
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Objectives

- The graph is undirected and connected
- Nodes dynamics:
  \[ \dot{p}_i = v_i, \quad \dot{v}_i = -v_i + u_i \]

Objectives

Design of event-based sampled controllers
- to achieve rendez-vous, i.e.
  \[ p_i(t) - p_j(t) \to 0 \text{ as } t \to \infty \quad \forall (i,j) \in I^2 \]
- to ensure the existence of a dwell-time per edge
Objectives

Challenges

- Analysis based on a weak Lyapunov function, i.e.
  \[ \dot{V} \leq 0 \quad \text{(and not } \dot{V} < 0) \]

- Local information only

Existing techniques: [Dimarogonas et al., 2012], [Nowzari & Cortes, 2012], [De Persis & Frasca, 2013], [De Persis, 2013] etc.
1. Introduction

2. Event-triggered controllers

3. Hybrid model

4. Analytical guarantees

5. Self-triggered controllers

6. Simulation results

7. Conclusions
Plan

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Approach

Emulation

- Design of the feedback laws while ignoring the resource constraints
- Synthesis of the triggering conditions
Feedback laws

**Feedback laws [Arcak, 2007]**

\( i \in \mathcal{I} \)

\[
\begin{align*}
u_i &= \sum_{j \in \mathcal{N}_i} \psi_{ij}(z_{ij}),
\end{align*}
\]

with

\[
\begin{align*}
z_{ij} &:= p_j - p_i
\end{align*}
\]

and

- \( \mathcal{N}_i \) is the set of neighbours of the node \( i \in \mathcal{I} \)
- \( \psi_{ij} \) is
  - continuously differentiable
  - nondecreasing
  - odd
  - \( \psi_{ij} = \psi_{ji} \) for \( i \in \mathcal{I} \) and \( j \in \mathcal{N}_i \)

\[
\begin{align*}
u_1 &= \psi_{12}(z_{12}) + \psi_{13}(z_{13}) + \psi_{14}(z_{14})
\end{align*}
\]
Feedback law (c’d)

Sampled feedback law

\[ u_i = \sum_{j \in \mathcal{N}_i} \psi_{ij}(\hat{z}_{ij}) \]

\( \hat{z}_{ij} \): sampled version of \( z_{ij} \)

\[ \dot{\hat{z}}_{ij} = 0, \quad \hat{z}_{ij}^+ = z_{ij} \]

One triggering condition per edge, e.g. \((i, j)\)
Triggering condition

Triggering condition \(\rightarrow\) we introduce \(\phi_{ij}\)

- Between two updates of \(\hat{z}_{ij}\)

\[
\dot{\phi}_{ij} = -\frac{1}{\sigma_{ij}} \left( 1 + \phi_{ij}^2 (\nabla \psi_{ij}(z_{ij}))^2 \right),
\]

\(\sigma_{ij} > 0\) to be designed

- Update when

\(\phi_{ij} = a_{ij}\)

then set

\(\phi_{ij}^{+} = b_{ij}\),

with \(0 \leq a_{ij} < b_{ij}\) to be designed
Triggering condition: example

**Example**

\[ u_1 = \psi_{12}(\hat{z}_{12}) + \psi_{13}(\hat{z}_{13}) + \psi_{14}(\hat{z}_{14}) \]

\[ \dot{\hat{z}}_{12} = 0 \quad \dot{\hat{z}}_{13} = 0 \quad \dot{\hat{z}}_{14} = 0 \]
Triggering condition: example

Example

\[ u_1 = \psi_{12}(\hat{z}_{12}) + \psi_{13}(\hat{z}_{13}) + \psi_{14}(\hat{z}_{14}) \]

\[ \dot{\hat{z}}_{12} = 0 \quad \dot{\hat{z}}_{13} = 0 \quad \dot{\hat{z}}_{14} = 0 \]
Triggering condition: example

**Example**

\[ u_1 = \psi_{12}(\hat{z}_{12}) + \psi_{13}(\hat{z}_{13}) + \psi_{14}(\hat{z}_{14}) \]

\[ \hat{z}_{12}^+ = z_{12} \quad \hat{z}_{13}^+ = \hat{z}_{13} \quad \hat{z}_{14}^+ = \hat{z}_{14} \]
Triggering condition: example

Example

\[ u_1 = \psi_{12}(\hat{z}_{12}) + \psi_{13}(\hat{z}_{13}) + \psi_{14}(\hat{z}_{14}) \]

\[ \dot{\hat{z}}_{12} = 0 \quad \dot{\hat{z}}_{13} = 0 \quad \dot{\hat{z}}_{14} = 0 \]
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Triggerring condition: example

Example

\[ u_1 = \psi_{12}(\hat{z}_{12}) + \psi_{13}(\hat{z}_{13}) + \psi_{14}(\hat{z}_{14}) \]

\[ \hat{z}_{12}^+ = \hat{z}_{12}, \quad \hat{z}_{13}^+ = \hat{z}_{13}, \quad \hat{z}_{14}^+ = z_{14} \]
Triggering condition: example

**Example**

\[
u_1 = \psi_{12}(\hat{z}_{12}) + \psi_{13}(\hat{z}_{13}) + \psi_{14}(\hat{z}_{14})
\]

\[
\dot{\hat{z}}_{12} = 0 \quad \dot{\hat{z}}_{13} = 0 \quad \dot{\hat{z}}_{14} = 0
\]
Triggering condition (c’d)

We need $\hat{z}_{ij} = -\hat{z}_{ji}$ and $\phi_{ij} = \phi_{ji}$.

Assumption

For any $(i,j) \in E$

(i) $a_{ij} = a_{ji}$, $b_{ij} = b_{ji}$, $\sigma_{ij} = \sigma_{ji}$.

(ii) $\hat{z}_{ij}$ and $\phi_{ij}$ are initialized at the same values as $-\hat{z}_{ji}$ and $\phi_{ji}$.
Interlude

Nonlinear sampled-data systems

\[ \dot{x}(t) = f(x(t), k(x(t_k))) \]

- Periodic sampling [Carnevale et al., 2007]
  \[ \dot{\phi} = -2L\phi - \gamma(\phi^2 + 1) \]

- Event-triggered control [Postoyan et al., 2011]
  \[ \dot{\phi} = -2L(x(t), x(t_k))\phi - \gamma(x(t), x(t_k)) \]
Plan

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Dynamics of node $i \in \mathcal{I}$

$$
\begin{align*}
\dot{p}_i &= v_i \\
\dot{v}_i &= -v_i + \sum_{j \in \mathcal{N}_i} \psi_{ij}(\hat{z}_{ij}) \\
\dot{\hat{z}}_{ij} &= 0 \\
\dot{\phi}_{ij} &= -\frac{1}{\sigma_{ij}} (1 + (\phi_{ij})^2 (\nabla \psi_{ij}(z_{ij}))^2) \\
P_i^+ &= P_i \\
V_i^+ &= V_i \\
\begin{pmatrix}
\hat{z}_{ij}^+ \\
\phi_{ij}^+
\end{pmatrix} &= 
\begin{cases}
\begin{pmatrix}
z_{ij} \\
b_{ij}
\end{pmatrix} & \phi_{ij} = a_{ij} \\
\begin{pmatrix}
\hat{z}_{ij} \\
\phi_{ij}
\end{pmatrix} & \phi_{ij} > a_{ij}
\end{cases}
\end{align*}
$$

Hybrid formalism of [Goebel et al., 2012]
Hybrid model

Dynamics of the overall system

\[
\begin{align*}
\dot{p} &= v \\
\dot{v} &= -v - D\Psi(\hat{z}) \\
\dot{\hat{z}} &= 0 \\
\dot{\phi} &= -\Sigma^{-1} \left( I + \left( \frac{\partial \Psi(z)}{\partial z} \Phi \right)^2 \right) \mathbf{1}
\end{align*}
\]

where

- \( p := (p_1, \ldots, p_N)^T \), \( v := (v_1, \ldots, v_N)^T \), \( \hat{z} := (\hat{z}_1, \ldots, \hat{z}_M)^T \), \( \phi := (\phi_1, \ldots, \phi_M)^T \)
- \( N \): number of nodes
- \( M \): number of edges
Jump map

\[
\begin{pmatrix}
\hat{z}^+ \\
\phi^+
\end{pmatrix} \in G(z, \hat{z}, \phi) \quad \exists \ell \quad \phi_\ell = a_\ell,
\]

For \( z, \hat{z}, \phi \in \mathbb{R}^M \),

\[
G(z, \hat{z}, \phi) := \{ G_\ell(z, \hat{z}, \phi) : \ell \in \{1, \ldots, M\} \text{ and } \phi_\ell = a_\ell \},
\]

with

\[
G_\ell(z, \hat{z}, \phi) := (\hat{z}_1, \ldots, \hat{z}_{\ell-1}, z_\ell, \hat{z}_{\ell+1}, \ldots, \hat{z}_M, \phi_1, \ldots, \phi_{\ell-1}, b_\ell, \phi_{\ell+1}, \ldots, \phi_M)^T
\]
Jump map (c’d)
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Theorem

Select $\sigma_1, \ldots, \sigma_M > 0$ s.t.

$$\max_{\ell \in \mathcal{E}_i} \sigma_\ell \leq \frac{1 - \varepsilon}{2 \deg_i}$$

$\varepsilon \in (0, 1)$, $\deg_i$: number of neighbours of agent $i$.

Then the following holds.

- The maximal solutions are precompact and approach the set

  $$\{(p, v, \hat{z}, \phi) : z = 0, \ v = 0, \ \phi_\ell \in [a_\ell, b_\ell] \text{ for } \ell \in \{1, \ldots, M\}\}.$$ 

- Solutions have a uniform semiglobal average dwell-time.

Recall: $\dot{\phi}_\ell = -\frac{1}{\sigma_\ell} \left(1 + \phi_\ell^2 \left(\nabla \psi_\ell(z_\ell)\right)^2\right)$
Uniform semiglobal average dwell-time

**Definition**

The solutions have a **uniform semiglobal average dwell-time** if \( \forall \Delta \geq 0, \exists \tau(\Delta) > 0 \) and \( n_0(\Delta) \in \mathbb{Z}_{>0} \) such that \( \forall \) solution \( \phi \) with \( |\phi(0, 0)| \leq \Delta \)

\[
k - i \leq \frac{1}{\tau(\Delta)}(t - s) + n_0(\Delta),
\]

for any \((s, i), (t, k) \in \text{dom } \phi \) with \( s + i \leq t + k \).

We say that the solutions have a **uniform global average dwell-time** when \( \tau \) and \( n_0 \) are independent of \( \Delta \).
Main steps of the proof

**System**

\[
\dot{q} = f(q) \quad \text{for } q \in C, \quad q^+ \in g(q) \quad \text{for } q \in D,
\]

1. Lyapunov function

\[
V := V_{\text{phys}} + V_{\text{cyber}}
\]

where

\[
V_{\text{phys}}(q) := \frac{1}{2} \sum_{i \in \mathcal{I}} v_i^2 + \sum_{\ell \in \{1, \ldots, M\}} \int_{0}^{z_\ell} \psi_\ell(s) ds
\]

\[
V_{\text{cyber}}(q) := \sum_{\ell \in \{1, \ldots, M\}} \phi_\ell \frac{1}{2} (\psi_\ell(\hat{z}_\ell) - \psi_\ell(z_\ell))^2.
\]

2. Lyapunov analysis

\[
\langle \nabla V(q), f(q) \rangle \leq -\varepsilon v^T v \quad \forall q \in C
\]

\[
V(g(q)) \leq V(q) \quad \forall q \in D,
\]

3. Maximal solutions are precompact

4. Solutions have a uniform semiglobal average dwell-time

5. Application of an hybrid invariance principle [Sanfelice et al., 2007]
Main steps of the proof

System

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Lyapunov function

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where

\[ V_{\text{phys}}(q) := \frac{1}{2} \sum_{i \in I} v_i^2 + \sum_{\ell \in \{1, \ldots, M\}} \int_0^{z_{\ell}} \psi_{\ell}(s) ds \]

\[ V_{\text{cyber}}(q) := \sum_{\ell \in \{1, \ldots, M\}} \phi_{\ell} \frac{1}{2} \left( \psi_{\ell}(\hat{z}_{\ell}) - \psi_{\ell}(z_{\ell}) \right)^2. \]

Lyapunov analysis

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V_{\text{cyber}}(q) := \sum_{\ell \in \{1, \ldots, M\}} \frac{1}{2} (\psi_\ell(\hat{z}_\ell) - \psi_\ell(z_\ell))^2.
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   \[ V(g(q)) \leq V(q) \quad \forall q \in D, \]

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Plan

1. Introduction
2. Event-triggered controllers
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6. Simulation results
7. Conclusions
Problem statement

- Event-triggered control

\[
\dot{\phi}_\ell = -\frac{1}{\sigma_\ell} \left(1 + \phi^2_\ell \left(\nabla \psi_\ell(z_\ell)\right)^2\right),
\]

\(\phi_\ell\) decreases from \(b_\ell\) to \(a_\ell\)

- Self-triggered control

\[
\dot{\phi}_\ell = -\frac{1}{\sigma_\ell} \left(1 + \phi^2_\ell \lambda_\ell\right),
\]

\(\lambda_\ell\) to be designed
Construction of $\lambda_\ell$

Design $\phi_\ell$ such that

$$|\psi_\ell(z)| \leq \bar{\psi} \quad \forall z \in \mathbb{R}$$

We have

$$\begin{align*}
\dot{z}_\ell &= \dot{p}_j - \dot{p}_i = v_j - v_i = w_\ell \\
\dot{w}_\ell &= \dot{v}_j - \dot{v}_i = -v_j + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - (v_i + \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik})) \\
&= -w_\ell + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik})
\end{align*}$$

We use the bounds

$$-(\deg_i + \deg_j)\bar{\psi} \leq \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik}) \leq (\deg_i + \deg_j)\bar{\psi}$$

to derive

$$z_\ell(t, k) \leq z_\ell(t, k) \leq \bar{z}_\ell(t, k),$$
Construction of $\lambda_\ell$

Design $\phi_\ell$ such that

$$|\psi_\ell(z)| \leq \bar{\psi} \quad \forall z \in \mathbb{R}$$

We have

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\begin{align*}
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&= -w_\ell + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik})
\end{align*}
\]

We use the bounds

\[-(\deg_i + \deg_j)\bar{\psi} \leq \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik}) \leq (\deg_i + \deg_j)\bar{\psi}\]

to derive

$$z_\ell(t, k) \leq z_\ell(t, k) \leq \bar{z}_\ell(t, k),$$
Construction of $\lambda_\ell$

Design $\phi_\ell$ such that

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We have

$$\dot{z}_\ell = \dot{p}_j - \dot{p}_i = v_j - v_i = w_\ell$$
$$\dot{w}_\ell = \dot{v}_j - \dot{v}_i = -v_j + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - (-v_i + \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik}))$$
$$= -w_\ell + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik})$$

We use the bounds

$$-(\deg_i + \deg_j)\overline{\psi} \leq \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik}) \leq (\deg_i + \deg_j)\overline{\psi}$$

to derive

$$z_\ell(t, k) \leq z_\ell(t, k) \leq \overline{z}_\ell(t, k),$$
Construction of $\lambda_\ell$

Design $\phi_\ell$ such that

$$|\psi_\ell(z)| \leq \bar{\psi} \quad \forall z \in \mathbb{R}$$

We have

$$\dot{z}_\ell = \dot{p}_j - \dot{p}_i = v_j - v_i = w_\ell$$
$$\dot{w}_\ell = \dot{v}_j - \dot{v}_i = -v_j + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - (-v_i + \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik}))$$

$$= -w_\ell + \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik})$$

We use the bounds

$$-(\deg_i + \deg_j)\bar{\psi} \leq \sum_{k \in \mathcal{N}_j} \psi_{jk}(\hat{z}_{jk}) - \sum_{k \in \mathcal{N}_i} \psi_{ik}(\hat{z}_{ik}) \leq (\deg_i + \deg_j)\bar{\psi}$$

to derive

$$\underline{z}_\ell(t, k) \leq z_\ell(t, k) \leq \overline{z}_\ell(t, k),$$
Construction of $\lambda_\ell$ (c’d)

$$\lambda_\ell(t, k) := \max_{z_\ell(t, k) \leq z \leq z_\ell(t, k)} (\nabla \psi_\ell(z))^2$$

When $\psi_\ell$ is such that $\nabla \psi_\ell$ is nonincreasing on $\mathbb{R}_{\geq 0}$

$$\lambda_\ell(t, k) := \begin{cases} 
(\nabla \psi_\ell(z_\ell(t, k)))^2 & \text{when } z_\ell(t, k) > 0 \\
(\nabla \psi_\ell(\overline{z}_\ell(t, k)))^2 & \text{when } \overline{z}_\ell(t, k) < 0 \\
(\nabla \psi_\ell(0))^2 & \text{when } z_\ell(t, k)\overline{z}_\ell(t, k) \leq 0 
\end{cases}$$
Main results

Similar hybrid model as for event-triggered control

Same guarantees:

- Rendez-vous is ensured
- Solutions have a semiglobal average dwell-time
Plan

1. Introduction
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3. Hybrid model
4. Analytical guarantees
5. Self-triggered controllers
6. Simulation results
7. Conclusions
Graph & parameters

Graph

Self-triggered controllers

Parameters

- $\psi_\ell : z \mapsto 10 \arctan z \ (\bar{\psi} = 10)$
- $\sigma_\ell = \frac{1-\varepsilon}{2\deg_i}, i \in \{1, \ldots, 5\}, \varepsilon = 1/4$
- $a_\ell = 0, b_\ell = b$ and different values of $b$
Figure
Table for different values of $b$

<table>
<thead>
<tr>
<th>$b$</th>
<th>$b = 1$</th>
<th>$b = 10$</th>
<th>$b = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average # edge events</td>
<td>2276</td>
<td>2101</td>
<td>2107</td>
</tr>
<tr>
<td>Average $t_{5%}$</td>
<td>13.62</td>
<td>15.14</td>
<td>15.86</td>
</tr>
</tbody>
</table>
Plan

1. Introduction
2. Event-triggered controllers
3. Hybrid model
4. Analytical guarantees
5. Self-triggered controllers
6. Simulation results
7. Conclusions
Conclusions

Summary

- Event-based sampled distributed controllers for a class of networked systems
- New type of triggering conditions
- Hybrid model a la [Goebel et al., 2012]
- Analysis based on an invariance principle for hybrid systems

Upcoming work

- Generalization of the node dynamics
- Prescriptive methodology for networked systems in general

Publications: MTNS 2014 (2 nodes), submitted to CDC 2014 (N nodes)