Symbolic Models for Control Systems

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Algorithmic controller synthesis from high level specifications:
Algorithmic controller synthesis from high level specifications:
Temporal Logic Specifications

• Linear temporal logic (LTL): wide variety of properties

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• LTL formula admits an equivalent (Büchi) automaton.
Algorithmic controller synthesis from high level specifications:
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The synthesis problem is hard because the model and the specification are heterogeneous.
Symbolic Model

Discrete system that is equivalent (bisimilar) to the continuous dynamics of the physical system:
Symbolic Model

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\[ \dot{x}(t) = f(x(t), u(t)) \]
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Discrete system that is equivalent (bisimilar) to the continuous dynamics of the physical system:

\[
\dot{x}(t) = f(x(t), u(t))
\]

\[
q(t^+) = g(q(t), x(t))
\]

\[
u(t) = k(q(t), x(t))
\]
Symbolic Model Approach to Control

A three step approach to controller synthesis.

Physical System:
\[ \dot{x}(t) = f(x(t), u(t)) \]

Controller:

Temporal Logic Specif.:
Symbolic Model Approach to Control

Step 1: Compute a symbolic model of the physical system.
Symbolic Model Approach to Control

Step 2: Algorithmic controller synthesis for the symbolic model.
Symbolic Model Approach to Control

Step 3: Refinement of the discrete controller.

Physical System:
\[ \dot{x}(t) = f(x(t), u(t)) \]

Hybrid Controller:
\[ q(t^+) = g(q(t), x(t)) \]
\[ u(t) = k(q(t), x(t)) \]

Temporal Logic Specif.:
Some Remarks

- The previous approach assumes exact equivalence (bisimilarity) between continuous and symbolic models.

- Unfortunately, the class of continuous (or hybrid) systems with known bisimilar symbolic models is quite restrictive:\[ \textit{timed automata, } o\text{-minimal systems, controllable linear systems}.\]
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• Unfortunately, the class of continuous (or hybrid) systems with known bisimilar symbolic models is quite restrictive: timed automata, o-minimal systems, controllable linear systems.

• A similar approach can be used based on approximately equivalent symbolic models.
Some Remarks

• The previous approach assumes exact equivalence (bisimilarity) between continuous and symbolic models.

• Unfortunately, the class of continuous (or hybrid) systems with known bisimilar symbolic models is quite restrictive: timed automata, o-minimal systems, controllable linear systems.

• A similar approach can be used based on approximately equivalent symbolic models.

• This relaxation allows us to extend the approach to several other classes of systems, in this talk:

  Approximately bisimilar symbolic models for switched systems.
Outline of the Talk

1. Switched systems and incremental stability

2. Approximate equivalence of dynamical systems:
   - Transition systems
   - Approximate bisimulation

3. Symbolic models for switched systems:
   - Construction of the symbolic model
   - Approximation result

4. Examples
Switched Systems

- A switched system $\Sigma$ consists of:
  - a state space $\mathbb{R}^n$;
  - a finite set of modes $P = \{1, \ldots, m\}$;
  - a set of switching signals $\mathcal{P} \subseteq S(\mathbb{R}^+, P)$; $S(\mathbb{R}^+, P)$: set of piecewise constant functions from $\mathbb{R}^+$ to $P$.
  - a collection of vector fields $F = \{f_p | p \in P\}$.

- $\Sigma_p$ denotes the continuous subsystem associated to $f_p$. 
Switched Systems

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  - a collection of vector fields $F = \{f_p \mid p \in P\}$.

- $\Sigma_p$ denotes the continuous subsystem associated to $f_p$.

- For switching signal $p \in \mathcal{P}$, state $x \in \mathbb{R}^n$, $x(t, x, p)$ denotes the associated trajectory of $\Sigma$:
  \[
  \dot{x}(t) = f_p(t)(x(t)), \quad x(0) = x.
  \]

- $x(t, x, p)$ denotes the trajectory of $\Sigma$ associated to constant switching signal $p(t) = p$. 
Stability of Switched Systems

- Switching between stable subsystems may create unstable behaviors:

- Stability ensured via:
  - Common Lyapunov function
  - Multiple Lyapunov functions + dwell time
The subsystem $\Sigma_p$ is *incrementally globally asymptotically stable* ($\delta$-GAS) if there exists a $\mathcal{KL}$ function $\beta_p$ such that for all $t \in \mathbb{R}^+$, for all $x, y \in \mathbb{R}^n$, the following condition is satisfied:

$$\|x(t, x, p) - x(t, y, p)\| \leq \beta_p(\|x - y\|, t)$$
δ-GAS Lyapunov Functions

A smooth function $V_p : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^+$ is a δ-GAS Lyapunov function for subsystem $\Sigma_p$ if there exist $K_\infty$ functions $\underline{\alpha}$, $\overline{\alpha}$ and $\kappa \in \mathbb{R}^+$ such that for all $x, y \in \mathbb{R}^n$:

$$\underline{\alpha}(\|x - y\|) \leq V_p(x, y) \leq \overline{\alpha}(\|x - y\|)$$

and

$$\frac{\partial V_p}{\partial x}(x, y)f_p(x) + \frac{\partial V_p}{\partial y}(x, y)f_p(y) \leq -\kappa V_p(x, y)$$

Theorem (Angeli 2002)

$\Sigma_p$ is δ-GAS if and only if it admits a δ-GAS Lyapunov function.
The switched system $\Sigma$ is *incrementally globally uniformly asymptotically stable* ($\delta$-GUAS) if there exists a $\mathcal{KL}$ function $\beta$ such that for all $t \in \mathbb{R}^+$, for all $x, y \in \mathbb{R}^n$, for all switching signals $p \in \mathcal{P}$, the following condition is satisfied:

$$\|x(t, x, p) - x(t, y, p)\| \leq \beta(\|x - y\|, t)$$

**Proposition**

*If there exists a common $\delta$-GAS Lyapunov function for subsystems $\Sigma_1, \ldots, \Sigma_m$, then the switched system $\Sigma$ is $\delta$-GUAS.*
Multiple $\delta$-GAS Lyapunov Functions

$S_{\tau_d}(\mathbb{R}^+, P)$ denotes the set of switching signals with dwell time $\tau_d$. The duration between two successive switching times is at least $\tau_d$.

Proposition

Let $\Sigma_{\tau_d} = (\mathbb{R}^n, P, \mathcal{P}, F)$ with $\mathcal{P} \subseteq S_{\tau_d}(\mathbb{R}^+, P)$. If for all $p \in P$, there exists a $\delta$-GAS Lyapunov function $V_p$ for subsystem $\Sigma_{\tau_d,p}$ and that in addition there exists $\mu \in \mathbb{R}^+$ such that:

$$\forall x, y \in \mathbb{R}^n, \forall p, p' \in P, \quad V_p(x, y) \leq \mu V_{p'}(x, y).$$

If $\tau_d > \frac{\log \mu}{\kappa}$, then $\Sigma_{\tau_d}$ is $\delta$-GUAS.
Supplementary Assumption

• In the following, we assume that there exists a $\mathcal{K}_\infty$ function $\gamma$ such that, for all $p \in P$

$$\forall x, y, z \in \mathbb{R}^n, \ |V_p(x, y) - V_p(x, z)| \leq \gamma(\|y - z\|).$$

• Working on a compact subset $C \subseteq \mathbb{R}^n$:

$$|V_p(x, y) - V_p(x, z)| \leq \left( \max_{p \in P, x, y \in C} \left\| \frac{\partial V_p}{\partial y} (x, y) \right\| \right) \|y - z\|. $$
Outline of the Talk

1. Switched systems and incremental stability

2. Approximate equivalence of dynamical systems:
   - Transition systems
   - Approximate bisimulation

3. Symbolic models for switched systems:
   - Construction of the symbolic model
   - Approximation result

4. Examples
Transition Systems

- Abstract description of discrete or continuous systems.

- A transition system $T$ consists of
  - a (discrete or continuous) set of states $Q$;
  - a (discrete or continuous) set of labels or actions $L$;
  - a transition relation $\subseteq Q \times L \times Q$;
  - a (discrete or continuous) set of outputs $O$;
  - an output map $H : Q \rightarrow O$.

$Q = \{\text{red, blue, green, yellow}\}$
$L = \{a, b\}$
$O = \{0, 1, 2\}$
Switched Systems as Transition Systems

• Consider a switched system $\Sigma = (\mathbb{R}^n, P, \mathcal{P}, F)$ with $\mathcal{P} = S(\mathbb{R}^+, P)$ and a time sampling parameter $\tau_s > 0$.

• Let $T_{\tau_s}(\Sigma)$ be the transition system where:
  • the set of states is $Q = \mathbb{R}^n$;
  • the set of labels is $L = P$;
  • the transition relation is given by
    \[ x \overset{p}{\rightarrow} x' \text{ iff } x(\tau_s, x, p) = x'; \]
  • the output set is $O = \mathbb{R}^n$;
  • the output function $H$ is the identity map over $\mathbb{R}^n$. 
Approximate Bisimulation

- Let $T_1, T_2$ be transition systems with a common set of labels $L$ and observed over a common metric space $(O, d)$.

- Let $\varepsilon \in \mathbb{R}^+$, a relation $R \subseteq Q_1 \times Q_2$ is an $\varepsilon$-approximate bisimulation relation if for all $(q_1, q_2) \in R$ :

  1. $d(H_1(q_1), H_2(q_2)) \leq \varepsilon$;
  2. $\forall q_1 \xrightarrow{l_1} q_1', \exists q_2 \xrightarrow{l_2} q_2'$, such that $(q_1', q_2') \in R$;
  3. $\forall q_2 \xrightarrow{l_2} q_2', \exists q_1 \xrightarrow{l_1} q_1'$, such that $(q_1', q_2') \in R$. 

\[ d(H_1(q_1), H_2(q_2)) \leq \varepsilon \]
Approximately Bisimilar Transition Systems

- $T_1$ and $T_2$ are *approximately bisimilar with precision* $\varepsilon$ if:
  1. For all $q_1 \in Q_1$, there exists $q_2 \in Q_2$, such that $(q_1, q_2) \in R$;
  2. For all $q_2 \in Q_2$, there exists $q_1 \in Q_1$, such that $(q_1, q_2) \in R$.

- Remark: if $\varepsilon = 0$, we recover the usual notion of “exact” bisimulation.
Approximately Bisimilar Transition Systems

• \( T_1 \) and \( T_2 \) are \textit{approximately bisimilar with precision} \( \varepsilon \) if:
  1. For all \( q_1 \in Q_1 \), there exists \( q_2 \in Q_2 \), such that \((q_1, q_2) \in R\);
  2. For all \( q_2 \in Q_2 \), there exists \( q_1 \in Q_1 \), such that \((q_1, q_2) \in R\).

• Remark: if \( \varepsilon = 0 \), we recover the usual notion of “exact” bisimulation.

Problem

\textit{Given a desired precision} \( \varepsilon > 0 \), compute a discrete transition system that is approximately bisimilar to \( T_{\tau_s}(\Sigma) \).
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Construction of the Symbolic Model

• We start by approximating the set of states $\mathbb{R}^n$ by:

$$[\mathbb{R}^n]_\eta = \left\{ q \in \mathbb{R}^n \mid q_i = k_i \frac{2\eta}{\sqrt{n}}, \ k_i \in \mathbb{Z}, \ i = 1, \ldots, n \right\},$$

where $\eta > 0$ is a state space discretization parameter.

• Approximation of the transition relation = “rounding”.

\[ x(\tau_s, q, p) \]

\[ q' \]
Construction of the Symbolic Model

• We define the discrete transition system $T_{\tau_s,\eta}(\Sigma)$ where:
  • the set of states is $Q = [\mathbb{R}^n]_\eta$;
  • the set of labels is $L = P$;
  • the transition relation is given by
    \[
    q \xrightarrow{p} q' \iff \|x(\tau, q, p) - q'\| \leq \eta;
    \]
  • the output set is $O = \mathbb{R}^n$;
  • the output map is given by $H(q) = q \in \mathbb{R}^n$.

• Are $T_{\tau_s}(\Sigma)$ and $T_{\tau_s,\eta}(\Sigma)$ approximately bisimilar?
  Yes, if $\Sigma$ is incrementally stable:
  Accumulation of successive “rounding errors” is contained by the incremental stability property.
Construction of the Symbolic Model

- We define the discrete transition system $T_{\tau_s, \eta}(\Sigma)$ where:
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- Yes, if $\Sigma$ is incrementally stable:

  Accumulation of successive “rounding errors” is contained by the incremental stability property.
Approximation Theorem

Theorem
Consider sampling parameters $\tau_s, \eta \in \mathbb{R}^+$ and a desired precision $\varepsilon \in \mathbb{R}^+$. Let us assume that there exists $V : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^+$ which is a common $\delta$-GAS Lyapunov function for subsystems $\Sigma_1, \ldots, \Sigma_m$. If

$$\eta \leq \min \left\{ \gamma^{-1} \left( (1 - e^{-\kappa \tau_s}) \alpha(\varepsilon) \right), \overline{\alpha}^{-1} \left( \alpha(\varepsilon) \right) \right\}$$

then, $T_{\tau_s}(\Sigma)$ and $T_{\tau_s,\eta}(\Sigma)$ are approximately bisimilar with precision $\varepsilon$. 

Any precision can be achieved!
Approximation Theorem

Theorem
Consider sampling parameters \( \tau_s, \eta \in \mathbb{R}^+ \) and a desired precision \( \varepsilon \in \mathbb{R}^+ \). Let us assume that there exists \( V : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^+ \) which is a common \( \delta \)-GAS Lyapunov function for subsystems \( \Sigma_1, \ldots, \Sigma_m \).

If
\[
\eta \leq \min \left\{ \gamma^{-1} \left( (1 - e^{-\kappa \tau_s}) \bar{\alpha}(\varepsilon) \right), \alpha^{-1} \left( \alpha(\varepsilon) \right) \right\}
\]
then, \( T_{\tau_s}(\Sigma) \) and \( T_{\tau_s,\eta}(\Sigma) \) are approximately bisimilar with precision \( \varepsilon \).

Any precision can be achieved!
Sketch of the Proof

Show that $R = \{(q_1, q_2) \in Q_1 \times Q_2 \mid V(q_1, q_2) \leq \alpha(\varepsilon)\}$ is an $\varepsilon$-approximate bisimulation relation:

- For $(q_1, q_2) \in R$, $\|q_1 - q_2\| \leq \alpha^{-1}(V(q_1, q_2)) \leq \varepsilon$.

- Let $q_1 \xrightarrow{l_1} q'_1$, then $q'_1 = x(\tau_s, q_1, l)$, let $q_2 \xrightarrow{l_2} q'_2$ then $\|x(\tau_s, q_2, l) - q'_2\| \leq \eta$ and

  $$V(q'_1, q'_2) \leq V(q'_1, x(\tau_s, q_2, l)) + \gamma(\eta)$$
  $$\leq V(x(\tau_s, q_1, l), x(\tau_s, q_2, l)) + \gamma(\eta)$$
  $$\leq e^{-\kappa \tau_s} V(q_1, q_2) + \gamma(\eta)$$
  $$\leq e^{-\kappa \tau_s} \alpha(\varepsilon) + \gamma(\eta) \leq \alpha(\varepsilon)$$
Case of Multiple Lyapunov Functions

Approximation result holds if we impose a dwell time $\tau_d$:

**Theorem**

Consider sampling parameters $\tau_s, \eta \in \mathbb{R}_+$, a desired precision $\varepsilon \in \mathbb{R}_+$ and a dwell time $\tau_d \in \mathbb{R}_+$. Assume that for all $p \in P$, there exists a $\delta$-GAS Lyapunov function $V_p$ for subsystem $\Sigma_{\tau_d,p}$. If $\tau_d > \frac{\log \mu}{\kappa}$ and

$$
\eta \leq \min \left\{ \gamma^{-1} \left( \frac{1}{\mu} \frac{1 - e^{-\kappa \tau_d}}{1 - e^{-\kappa \tau_s}} \alpha(\varepsilon) \right), \alpha^{-1}(\alpha(\varepsilon)) \right\}
$$

then, $T_{\tau_s}(\Sigma_{\tau_d})$ and $T_{\tau_s,\eta}(\Sigma_{\tau_d})$ are approximately bisimilar with precision $\varepsilon$. 

Bound on the dwell time is the same as in the $\delta$-GAS theorem!
Case of Multiple Lyapunov Functions

Approximation result holds if we impose a dwell time $\tau_d$:

**Theorem**

Consider sampling parameters $\tau_s, \eta \in \mathbb{R}^+$, a desired precision $\varepsilon \in \mathbb{R}^+$ and a dwell time $\tau_d \in \mathbb{R}^+$. Assume that for all $p \in P$, there exists a $\delta$-GAS Lyapunov function $V_p$ for subsystem $\Sigma_{\tau_d,p}$.

If $\tau_d > \frac{\log \mu}{\kappa}$ and

$$
\eta \leq \min \left\{ \gamma^{-1} \left( \frac{1}{\mu - e^{-\kappa \tau_d}} \left( 1 - e^{-\kappa \tau_s} \right) \alpha(\varepsilon) \right), \bar{\alpha}^{-1}(\alpha(\varepsilon)) \right\}
$$

then, $T_{\tau_s}(\Sigma_{\tau_d})$ and $T_{\tau_s,\eta}(\Sigma_{\tau_d})$ are approximately bisimilar with precision $\varepsilon$.

*Bound on the dwell time is the same as in the $\delta$-GAS theorem!*
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4. Applications
• Power converter with switching control:

\[ x(t) = [i_l(t), v_c(t)]^T. \]

• Control objective: regulate the output voltage.

*Formulated as a safety property: \( \square S \).
DC-DC Converter

• Dynamics of the system:

\[
\dot{x}(t) = A_p x(t) + b, \quad p = 1, 2
\]

where

\[
A_1 = \begin{bmatrix}
-\frac{r_i}{x_i} & 0 \\
0 & -\frac{1}{x_c} \frac{1}{r_0+r_c}
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
-\frac{1}{x_i} \left(\frac{r_i}{r_0+r_c} + \frac{r_0}{r_0+r_c}\right) & -\frac{1}{x_i} \frac{r_0}{r_0+r_c} \\
\frac{1}{x_c} \frac{r_0}{r_0+r_c} & -\frac{1}{x_c} \frac{1}{r_0+r_c}
\end{bmatrix}, \quad b = \begin{bmatrix}
\frac{v_s}{x_i} \\
0
\end{bmatrix}.
\]

• Existence of a common \( \delta \)-GAS Lyapunov function of the form:

\[
V(x, y) = \sqrt{(x - y)^T M(x - y)}.
\]
Symbolic Model of the DC-DC Converter

(Useless) symbolic model: \( \tau_s = 0.5, \, \eta = \frac{1}{40\sqrt{2}}, \, \varepsilon = 2.6. \)
Symbolic model: $\tau_s = 0.5$, $\eta = \frac{1}{4000\sqrt{2}}$, $\varepsilon = 0.026$ (642001 states!).

Supervisory controller

Trajectory of the switched system
A Second Example

- Dynamics of the system:

\[ \dot{x}(t) = A_p x(t) + b_p, \quad p = 1, 2. \]

where

\[ A_1 = \begin{bmatrix} -0.25 & 1 \\ -2 & -0.25 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.25 & 2 \\ -1 & -0.25 \end{bmatrix}, \]

\[ b_1 = \begin{bmatrix} -0.25 \\ -2 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0.25 \\ 1 \end{bmatrix}. \]

- Multiple \( \delta \)-GAS Lyapunov functions with dwell time \( \tau_d = 2 \):

\[ V_p(x, y) = \sqrt{(x - y)^T M_p (x - y)}, \quad M_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}. \]
Symbolic model: $\tau_s = 0.5$, $\eta = \frac{1}{100\sqrt{2}}$, $\varepsilon = 0.34$ (7696008 states!).

Control actions when mode=1 and dwell time has elapsed

Control actions when mode=2 and dwell time has elapsed
Controller for the Symbolic Model (II)

Using a lazy control strategy:
Control of the Switched System

Switching signal and trajectory

Trajectory of the switched system
Summary

- Approximately bisimilar symbolic models are rigorous tools for controller synthesis:
  - High level specifications in temporal logics.
  - Controllers are correct by design.
- They are computable for interesting classes of systems:
  - Switched systems, linear or nonlinear continuous systems.
  - Any precision can be achieved.
- Incremental stability needed.
- Symbolic models have potentially a huge number of states.
- They allow to leverage techniques from discrete systems to continuous and hybrid systems.
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Future Work in the VEDECY Project

• Multiscale and adaptive symbolic models:
  • Non-uniform precision allowing a global reduction of the number of states in the symbolic models.
  • Non-deterministic symbolic models.
  • Only approximate simulation.

• On the fly computation of multiscale symbolic models:
  • During controller synthesis.
  • Local adaptation of precision: abstraction refinement.

• Complexity reduction of synthesized controllers:
  • Structural techniques: dynamic to static controller.
  • Geometric and topological techniques: efficient encoding.

• Development of a tool (collaboration PESSOA).
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