On the Existence of a Unique Local Solution to the Hasegawa-Mima Equation in Periodic Sobolev Spaces

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Abstract

The Hasegawa-Mima PDE is a simplified, yet highly non-linear, 2D turbulent system model which describes the time evolution of drift waves caused during magnetic plasma confinement.

In this talk, we represent and consider HM as a system of linear initial value Hyperbolic-Elliptic PDEs

$$\begin{cases} w_t + \vec{V}(u) \cdot \nabla w = k u_y \\ -\Delta u + u = w \end{cases}$$

on rectangular domain $\Omega \times (0, T]$ with some periodic boundary conditions, where $\vec{V}(u) = -u_y\,\vec{\mathbf{i}} + u_x\,\vec{\mathbf{j}}$ is a divergence-free vector field.

After properly defining Periodic Sobolev Spaces $H_P^m(\Omega)$, we formulate HM as the limit of a sequence of fixed-point Petrov-Galerkin problems, and show that, for the initial data $u(0) \in H_P^3(\Omega)$ with $w(0) := \Delta u(0) + u(0) \in H_P^1(\Omega) \cap L^\infty(\Omega)$, we have a unique local solution pair (u, w) with regularities

$$\begin{cases} u \in C([0,T], H_P^2(\Omega)) \cap L^\infty(0,T; H_P^3(\Omega)) \\ u_t \in L^\infty(0,T; H_P^2(\Omega)) \\ w \in C([0,T], L^2(\Omega)) \cap L^\infty(0,T; H_P^1(\Omega)) \\ w_t \in L^\infty(0,T; L^2(\Omega)) \end{cases}$$

In contrast to proofs found in literature that use semigroup methods for $u_0 \in H^4(\Omega)$, our approach provides a Finite Element scheme for simulating the solution.

Keywords: Drift Waves; Hasegawa-Mima; Periodic Sobolev Spaces; Petrov-Galerkin; Schauder fixed point.