Parameter Estimation Through Strucutrue-Preserving Approximate Bayesian Computation for Stochastic Neural Mass Models

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Abstract

In this talk we perform statistical inference for a stochastic neural mass model using approximate Bayesian computation. Stochastic neural mass models are used to describe the electrical activity of a whole population of neurons with average properties, and have been reported to reproduce, for example EEG/MEG/SEEG data. Here we focus on a specific reformulation of the Jansen and Rit neural mass model [1] as a stochastic differential equation (JR-SDE) with additive noise [2]. We can analyse this new stochastic version of the model through its dynamical and structural properties. In particular, we are interested in estimating some parameters that have been shown to be relevant for the description of α-rhythmic and epileptic behaviour.

In [2], the authors declared that the JR-SDE can be re-formulated as a stochastic Hamiltonian equation, which enabled them to prove its ergodicity. This guarantees that the distribution of the 6-dimensional solution process $X(t) = (X_0(t), ..., X_5(t))^T$, $t \in [0, T]$ converges exponentially fast towards a unique invariant measure and allows us to extract important statistical properties from single sample paths.

Here we perform statistical inference for this stochastic model, making use of a numerical splitting scheme that has been shown to preserve the structural model properties, differently from commonly used schemes, such as the Euler Maruyama method [2]. From an experimental point of view, the solution process $(X(t))_{t\in[0,T]}$ is partially observed through the EEG-related stochastic process $Y(t) = X_1(t) - X_2(t)$, $t \in [0, T]$. Two main difficulties arise: First, due to the fact that the non-linear and multi-dimensional SDE cannot be explicitly solved, the dynamics of the signal process $(Y(t))_{t\in[0,T]}$ can be only simulated through the numerical scheme.

Second, the corresponding underlying likelihood function is intractable. We tackle this last issue by considering the likelihood-free and simulation based approximate Bayesian computation (ABC) approach [3]. This is a Bayesian technique that necessitates plenty of synthetic data simulations from the original model.

In the proposed statistical analysis, the crucial part is to define reliable distance criteria to successfully compare the simulated synthetic signals with the observed reference data. Due to the large variability in the data generated by the JR-SDE, neither the calculation of distances between the data itself nor the use of standard summary statistics work. Even more sophisticated and common distances for time series fail. To overcome this difficulty, we propose to transform the signal data from time to frequency domain by considering the corresponding spectral density. The spectral density depends on parameters that directly affect the frequency as well as the amplitude and, therefore, carries significant dynamical and structural information.

By this clever use of the parameter dependent structural and dynamical properties of the system, the ABC approach, with the adopted numerical splitting method, enables us to fit the model to real EEG data.

References

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