# Semidefinite Approximations of Invariant Measure Supports and Reachable Sets for Discrete-time Polynomial Systems

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Given a compact semialgebraic set X0 included in $R^n$, a polynomial map f from $R^n$ to $R^n$, we consider the problem of approximating the union $X*$ of $Xinv$ and $Xinfty$, where $Xinv$ is the maximal support of Lebesgue dominated invariant measures w.r.t. f and $Xinfty$ is the reachable set for the discrete-time polynomial system $x_{t+1} = f(x_t)$, $t \in N$, $x0 \in X0$. Assuming that $X*$ is included in X, with X being a "simple" set (box or ellipsoid), we provide a method to compute certified outer approximations of $X*$.
Here, the proposed method consists of building a hierarchy of relaxations for the infinite dimensional moment problem whose optimal value is the volume of $X*$ and whose optimum is the restriction of the Lebesgue measure on $X*$. Then, one can outer approximate $X*$ as closely as desired with a hierarchy of super level sets of the form $X^r := xinX : vr(x) \geq 0$, for some polynomials vr of increasing degrees 2 r. For each fixed r, finding the coefficients of the polynomial vr boils down to computing the optimal solution of a semidefinite program. We provide guarantees of strong convergence to $X*$ in $L1(X) - norm$, when the degree of the polynomial approximation tends to infinity and prove in particular that $lim[vol(X^r X*)] = 0$ when r tends to infinity. We also present some application examples together with numerical results.