Abstract

Consider the classical stabilizing state feedback design in the linear dynamical system

\[ \dot{x} = Ax + Bu, \]

subject to performance specifications with an additional requirement that the control input vector

\[ u = Kx \]

has as many zero entries as possible. Here, \( x = x(t) \in \mathbb{R}^n \) is the vector of phase variables, \( u \in \mathbb{R}^m, m < n \), and \( A, B, K \) are constant matrices of appropriate dimensions.

The desired gain matrix \( K \) will be referred to as a \textit{sparse controller}. We discuss a straightforward approach to solving this kind of nonconvex control problems in an approximate manner by introducing the associated \textit{convex surrogate}. This surrogate problem is the minimization of a properly chosen matrix norm subject to LMI constraints.

The novelty of the approach is

(i) the problem formulation itself and

(ii) the construction of the convex surrogate.

We will show that this approach is equally easily applies to other problems such as state feedback design from incomplete state vector, design of low-dimensional output to be used in static output feedback, linear quadratic design, to name just a few.

The results of preliminary numerical experiments are twofold. First, it turns out, that in many test problems, the number of controls indeed can be considerably reduced without significant loss in performance. Second, the number of nonzero entries obtained with our method is either very close to or coincide with the minimum possible amount. The approach can be further extended to handle numerous problems of optimal and robust control in sparse formulation.