Report from Dagstuhl Seminar 11211

Geometric Modeling

Edited by

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- Abstract

This report documents the program and the results of Dagstuhl Seminar 11211 "Geometric Modeling", taking place May 22-27 2011. The focus of the seminar was to discuss modern and emerging topics in Geometric Modeling by researchers and industrial scientists from all over the world.

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1 Executive Summary

Thomas Grandine Stefanie Hahmann Jörg Peters Wenping Wang

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The 8th Dagstuhl seminar was attended by 51 leading researchers coming from 4 continents and 21 countries (!). This high number shows the strong interest of the community for this event. The feedback from participants was very positive. A total of 45 presentations were grouped together into 12 sessions and let a lot of room for stimulating and fruitful discussions in the harmonic Dagstuhl atmosphere.

Geometric Modeling is the branch of Computer Science concerned with the acquisition, representation, modeling and analysis of 3-dimensional geometry. While its combination of technically complex and often interdisciplinary approaches is grounded both in applied mathematics and computer science data structures and theory, applications of the field therefore cover a wide collection of areas from classical product design, virtual prototyping and simulation to computer graphics, scientific visualization, medical imaging, multimedia and entertainment. It is therefore fitting that the seminar was attended by 7 leading scientists and engineers from industry.

The presentations ranged from surface reconstruction, GPU programming, to curve and surface modeling with classical splines, surface meshes and new subdivision methods based

on algebraic and differential geometry methods with applications in medical or architectural modeling and in videao and gaming industry. As with all previous Dagstuhl Seminars on Geometric Modeling, the conference proceedings will be published as a special issue in an international journal.

A special event during the conference was the John Gregory Memorial Award honoring Carl de Boor, Malcolm Sabin and Gershon Elber. This award is presented every three years at Dagstuhl and honors fundamental contributions to the field of geometric modeling.

The organizers thank all the attendees for their contributions and extend special thanks to the team of Schloss Dagstuhl for helping to make this workshop a success. As always, we enjoyed the warm atmosphere of the Schloss, which supports both formal presentations as well as informal exchanges of ideas.

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3 Overview of Talks

3.1 An intuitive way for constructing parametric quadric triangles

Gudrun Albrecht (University of Valenciennes, FR)

We present an intuitive algorithm for constructing rational triangular quadratic patches which lie on quadrics. The input of the algorithm is three vertex data points in 3D and normals at these points. It emanates from two existing methods thanks to an interesting geometric relation connecting them.

The sufficient condition for a configuration of vertex and normal data to allow for the existence of a rational triangular quadratic patch lying on a quadric whose tangent planes at the vertices are those prescribed by the given normals is the concurrence of certain cevians. When these conditions are not met we offer an optimization procedure to tweak the normals, without varying the vertex data, so that for the new normals there is a rational triangular quadratic patch that lies on a quadric.

3.2 Geometric Modeling and Analysis of Protein Complexes using algebraic splines

Chandrajit Bajaj (Univ. of Texas at Austin, US)

The rate limiting step of atomistic molecular dynamics (and binding affinity estimations in disease therapeutics), is the accurate evaluation of the molecular configurational potential energy E, and the vector atomic derivatives of E (a.k.a. force fields). In this talk I shall first introduce an implicit solvent free-energy model of protein complexes based on either Generalized Born, and Poisson-Boltzmann formulations, and present analysis algorithms for the rapid computation of bound and unbound free-energies. Our new algorithms rely on a novel algebraic surface spline element and the use of irregular fast Fourier methods, for capturing both the geometry of molecular solvation interfaces, as well as the efficient computation of polarization energies/forces.

3.3 A Volume Approach to Model Repair and Smoothing

Pere Brunet (Universitat Politècnica de Catalunya – Barcelona, ES)

In this talk we propose a novel strategy to automatically segment volume medical data using a high-quality mesh segmentation of a "example" model as a guiding example. The example mesh is deformed until it matches the relevant volume features. The algorithm starts from a volume model (scalar field of densities) to be segmented, together with an already existing

segmentation (polygonal mesh) of the same organ, usually from a different person. The preprocess step computes a suitable attracting scalar field in the volume model. After an approximate 3D registration between the example mesh and the volume (this is the only step requiring user intervention), the algorithm works by minimizing an energy and adapts the shape of the polygonal mesh to the volume features in order to segment the target organ.

3.4 Snake based segmentation of teeth from digitized dental casts

Guido Brunnett (TU Chemnitz, DE)

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Joint work of Guido Brunnett, Thomas Kronfeld, David Brunner
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During the past years several innovative and technological developments have been made in oral surgery. Today, digitized dental casts are common for simulation and planning of orthodontic interventions. In order to work with these models, knowing the exact position of the teeth is of high importance. In this paper, we present a new method for tooth segmentation with minimal user interaction. At the beginning, an initial estimate for the separating curve between the teeth and the gum is computed and optimized by use of an active contour. The second step calculates the dental arch and the interstices between the teeth. In order to detect each tooth surface exactly, we finally position a snake around the cusp of each tooth.

3.5 Adaptive Finite Element Methods for Elliptic Equations using PHT-Splines

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Falai Chen (Univ. of Science & Technology of China – Anhui, CN)
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Joint work of Falai Chen, Li Tian, Qiang Du
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In this talk, we propse to use the polynomial splines over hierarchical T-meshes (PHT-splines) to solve elliptic equations with adaptive finite element methods. We develop a residual-based a posteriori error estimate for the finite element discretization of elliptic equations and establish their approximation properties. In addition, we conduct numerical experiments to verify the theoretical results and demonstrate the robustness of the error estimate and the effective approximations provided by the new spline space.

3.6 Computing with A B-spline like basis for the Powell Sabin 12-split

Elaine Cohen (University of Utah - Salt Lake City, US)

The Powell Sabin 12-split has the attribute that it can be used as a macro-element over arbitrary triangulations to create C1 smooth surfaces. We present attributes of a new B-spline like basis based on simplex splines and demonstrate applications.

3.7 Representing Non-Manifold Simplicial Shapes in Arbitrary Dimensions

Leila De Floriani (University of Genova, IT)

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Simplicial complexes are commonly used to describe non-manifold shapes in several applications, including finite element analysis, solid modeling, animation, terrain modeling and visualization of scalar and vector fields. In our work, we have developed several data structures for non-manifold simplicial shapes discretized as d-dimensional simplicial Euclidean complexes, both dimension-specific, and, more recently, dimension-independent. We will present the dimension-independent data structures, analyze and compare them, also with other representations, based on expressive power, storage requirements, and effectiveness in supporting navigation and updates. We will present our dimension-independent library for encoding and manipulating Euclidean simplicial complexes based on such data structures, which will soon be released in the public domain. We will finally discuss issues on topological modeling of non-manifold shapes.

3.8 A Sketching Interface for Feature Curve Recovery of Free-Form Surfaces

Ellen Dekkers (RWTH Aachen, DE)

In this talk, we present a semi-automatic approach to efficiently and robustly recover the characteristic feature curves of a given free-form surface where we do not have to assume that the input is a proper manifold. The technique supports a sketch-based interface where the user just has to roughly sketch the location of a feature directly on the input mesh. The system then snaps this initial curve to the correct position based on a graph-cut optimization scheme that takes various surface properties into account. Additional position constraints can be placed and modified manually which allows for an interactive feature curve editing functionality. We apply our technique to two practical scenarios. At first, feature curves can be used as handles for intuitive deformation of non-manifold surfaces. Secondly, we consider a practical problem in reverse engineering where we generate a statistical (PCA) shape model for car bodies.

3.9 Locally Refined B-splines

Tor Dokken (SINTEF - Oslo, NO)

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Isogeometric analysis (IGA), introduced in 2005 by T.J. R Hughes, replaces traditional Finite Elements with element descriptions using tensor product 3-varaite NonUniform Rational

B-splines (NURBS), thus combining the exact topology description in Finite Element Analysis with the accurate shape representation in Computer Aided Design. NURBS based IGA shows great promise; however, NURBS lack the necessary local refinement needed. Locally Refined splines (LR B-splines), such as T-splines and the novel Locally Refined B-splines (LR B-splines) show great promise with respect to the needed local refinement. The talk will address IGA, and LR B-splines, and report on the work on the theoretical basis of LR B-splines.

3.10 Reconstruction of 3D objects from 2D cross-sections by the 4-point subdivision schemes

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Nira Dyn (Tel Aviv University, IL)
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We adapt the 4-point subdivision schemes to refine 2D sets, by expressing it in terms of repeated binary averages, also with negative small weights, and by using a newly designed affine combination of sets. The resulting set refining subdivision scheme is convergent and the limits generated by it approximates objects when the scheme is applied to their cross-sections. The quality of the approximation depends on the continuity properties of the sampled object. Several examples of reconstructed objects by this method will be demonstrated.

3.11 Class-A surfacing in practice

Matthias Eck (Dassault Systémes Deutschland GmbH - Hannover, DE)

Class-A surfacing characterizes since decades now the creation of aesthetic surfaces with highest visual expectations. We briefly sketch the main industrial application areas, some simple but important rules for the creation, and how software tools can support that. After outlining some exemplary actual requests, we consider next-generation possibilities including associative Class-A surfacing and tighter process integration with Concept Modeling.

3.12 Optimal Ruled Surface Fitting

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Gershon Elber (Technion - Haifa, IL)
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Ruled surfaces play a major role in many manufacturing processes, such as wire EDM and side CNC milling. In this work, we explore an optimal ruled surface fitting scheme to a given general freeform rational surface. S.

The solution is divided into two parts. In the first and given a line segment that spans two (boundary) points on S, we derive a scheme that efficiently computes the maximal distance from the line segment to S. Then, and using a discrete sampled set of boundary points on S,

all the possible line segments between point-pairs are considered and the best ruling fit is derived using dynamic-programming.

The result is an optimal ruling fit for general hyperbolic regions that is less effective on convex domains, a concern we also address in the examples section.

3.13 Agnostic G1 Gregory Surface Fitting

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Gerald Farin (ASU - Tempe, US)
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Joint work of Gerald Farin, Ashish Amresh
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We utilize a geometric formulation of G1 conditions between adjacent Gregory patches. These conditions work equally well for three- or four- sided patches: they are "agnostic" to the patch type. We thus arrive at a simple and geometric framework for generating mixed patch type surfaces.

3.14 Spatial Pythagorean hodographs, quaternions, and rotations in \mathbb{R}^3 and \mathbb{R}^4

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Rida T. Farouki (Univ. of California – Davis, US)
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Quaternions, the first example of a non-commutative algebra, arose as a by-product of Hamilton's failed attempt to construct an "algebra of triples". Hamilton envisaged the quaternions as the "new language" of science and technology, but their place was usurped by vector analysis, an algebraically crude and overtly pragmatic subset of the quaternion algebra. A simple quaternion expression automatically generates Pythagorean quartuples of polynomials, thus yielding an elegant rotation invariant characterization of Pythagorean hodographs in \mathbb{R}^3 . Quaternions provide compact and intuitive descriptions for rotations in \mathbb{R}^3 , a fact that has lead to a renewed interest in them for robotics, computer graphics, animation, and related fields. Quaternions also describe rotations in \mathbb{R}^4 , whose strange properties provide a cautionary tale against extrapolating our geometric intuition from \mathbb{R}^2 and \mathbb{R}^3 to Euclidean spaces of higher dimension.

3.15 Quantum Bezier Curves

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Ron Goldman (Rice University - Houston, US)
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Joint work of Ron Goldman, Plamen Simeonov, Vasilis Zafiris
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Quantum Bezier curves are an extension of standard Bezier curves based on the quantum calculus. To study these new quantum Bezier curve, we extend standard blossoming (polar forms) to quantum blossoming. We apply quantum blossoming to derive extensions of

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well known algorithms for standard Bezier curves, including de Casteljau's algorithms for recursive evaluation and subdivision, to quantum Bezier curves. We also derive algorithms for quantum differentiation of quantum Bezier curves as well as new quantum versions of Marsden's identity.

3.16 The Convolution of two B-splines

Thomas Grandine (The Boeing Company - Seattle, US)

An important application has arisen at The Boeing Company for which spline convolution appears to be a remarkably effective solution. Although the literature on this topic is fairly extensive, most of it is devoted to very special cases, e.g. approximation with cardinal splines, which is not directly applicable to The Boeing problem. One old result, however, did prove to be unexpectedly relevant to this problem. This talk will review that result and show some numerical results of that old technique applied to a spline convolution problem related to solar energy production.

3.17 Parametrisation in Isogeometric Analysis

Jens Gravesen (Technical University of Denmark, DK)

One of the attractive features of isogeometric analysis is the exact representation of the geometry. The geometry is furthermore given by a relative low number of control points and this makes isogeometric analysis an ideal basis for shape optimisation.

One problem is that the geometry of the shape is given by the boundary alone. And, it is the boundary and its parametrisation that is changed by the optimisation procedure. But isogeometric analysis requires a parametrisation of the whole domain. So in every optimisation cycle we need to extend a parametrisation of the boundary of a domain to the whole domain. It has to be fast in order not to slow the optimisation down but it also has to be robust and give a parametrisation of high quality. These are conflicting requirements so we propose the following approach. During the optimisation a fast linear method is used, but if the parametrisation becomes singular or close to singular then the optimisation is stopped and the parametrisation is improved using a nonlinear method. The optimisation then continues using a linear method.

We will explain how the validity of a parametrisation can be checked and we will describe various ways to parametrise a domain. We will in particular study the Winslow functional which turns out to have some desirable properties.

3.18 Realistic-Looking Wrinkles added to Coarse Cloth Simulation

Stefanie Hahmann (University of Grenoble – LJK-INRIA, FR)

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Joint work of Stefanie Hahmann, D. Rohmer, M.-P. Cani, A. Sheffer, T. Poppa
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Moving garments and other cloth objects exhibit dynamic, complex wrinkles. Generating such wrinkles in a virtual environment currently requires either a time-consuming manual design process, or a computationally expensive simulation, often combined with accurate parameter-tuning requiring specialized animator skills. Our work presents an alternative approach for wrinkle generation which combines coarse cloth animation with a post-processing step for efficient generation of realistic-looking fine dynamic wrinkles. Our method uses the stretch tensor of the coarse animation output as a guide for wrinkle placement. To ensure temporal coherence, the placement mechanism uses a space-time approach allowing not only for smooth wrinkle appearance and disappearance, but also for wrinkle motion, splitting, and merging over time. Our method generates believable wrinkle geometry using specialized curve-based implicit deformers. The method is fully automatic and has a single user control parameter that enables the user to mimic different fabrics.

3.19 Generalization of the incenter subdivision scheme

Victoria Hernandez-Mederos (ICIMAF – La Habana, CU)

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In this talk we present an interpolatory Hermite subdivision scheme depending on a free parameter, which generalizes in certain way Deng's incenter subdivision scheme (CAGD, 2010). We prove that for any value of the free parameter the limit curve is G1 continuous. Moreover, if the vertices of the initial polygon and the tangent vectors are sampled from a circle with any arbitrary spacing, then the subdivision curve is the circle. The proposed scheme is shape preserving, avoids undesirable oscillations of the subdivision curve and introduces inflection points only in those regions of the curve where the control polygon suggests a change of convexity. Several examples are presented to demonstrate the performance of the scheme and we formulate some conjectures supported by numerical experiments.

3.20 Geometric Subdivision Curves

Kai Hormann (Universität Lugano, CH)

While linear subdivision schemes for curves and surfaces are well-understood by now, we still have but a few tools for analyzing the non-linear setting, although non-linear schemes often yield better results than their linear counterparts. In this talk we discuss a geometric condition which guarantees that a non-linear interpolating subdivision scheme produces

tangent continuous limit curves. As an example, we present two non-linear variants of the classical 4-point scheme which satisfy this condition.

3.21 Topological Classification of Intersections of Two Ring Tori

Xiaohong Jia (University of Hong Kong, HK)

We provide an enumeration for the topology of the intersection curve of two ring tori in generic positions and orientations. Based on the enumeration, a simple algebraic approach of deciding the topology of the intersection curve of two ring tori is presented, which requires only counting the real root number of two univariate quartic polynomials constructed from the implicit equations of these two ring tori.

3.22 Parameterization of Contractible Domains Using Sequences of Harmonic Maps

Bert Jüttler (University of Linz, AT)

We propose a new method for parameterizing a contractible domain (called the computational domain) which is defined by its boundary. Using a sequence of harmonic maps, we first build a bijective mapping from the computational domain to the parameter domain, i.e., the unit square or unit cube. Then we parameterize the original domain by a spline approximation of the inverse mapping. Numerical simulations of our method were performed with several shapes in 2D and 3D to demonstrate that our method is suitable for various shapes. The method is particular useful for isogeometric analysis because it provides an extension from a boundary representation of a model to a volume representation.

3.23 Constructing Smooth Branching Surfaces from Parallel Cross Sections

Panagiotis Kaklis (National TU - Athens, GR)

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Joint work of P. Kaklis, M. Karavelas, A.-A. Ginnis
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We deal with the problem of constructing G^1 surfaces that interpolate data points on parallel cross sections, consisting of simple disjoined and non-nested contours, the number of which varies from plane to plane. We shall present preliminary results towards a more stable and general approach, versus that developed in [1] and [2]. In our novel approach we modify the way the "surrounding curve" is constructed, and we generalize our approach so as to be able to incorporate in the construction contours that are away from the convex hull of contours

(which was not handled in [1] and [2]). These modifications/generalizations affect a crucial step in our smooth branching surface framework, namely the treatment of the "one-to-many" configuration. The approach is, so far, limited to convex contours and is based on the Voronoi diagram / Delaunay graph of each coplanar contour-set. The Voronoi diagram/Delaunay graph of the contours is used for deciding:

- 1. membership and order of the contours touching the surrounding curve (via proximity to the convex-hull boundary),
- 2. (combinatorial) description of the surface patches to be created away from the surrounding curve (via neighboring information provided by the Voronoi diagram), and
- 3. location of the surface saddle areas (via immersing Voronoi vertices). The approach is completed by introducing a methodology for the construction of:
 - \blacksquare the (G^1) surrounding curve,
 - the (G^1) guiding curves along with tangent ribbons defined on them and finally,
 - a patchwork of locally defined quadrilateral Gordon-Coons patches that define the final surface.

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3.24 Rational splines for free-form design with basic shapes

Rimvydas Krasauskas (Vilnius University, LT)

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Joint work of Rimvydas Krasauskas, Jorg Peters
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We present a rational G^2 (curvature continuous) analogue of non-uniform C^2 cubic B-spline paradigm. These rational splines can exactly reproduce the parts of multiple basic shapes, such as cyclides and quadrics in one smoothly connected structure.

3.25 Construction of spline basis over quad-linear sub-domains with T-junctions

Tae-Wan Kim (Seoul Nat. University, KR)

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Joint work of Tae-Wan Kim, Yeong-hwa Seo, Bernard Mourrain
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We give a talk about the existence of a partition of unity basis and the method of getting that basis having local support property in the case of local domain subdivision with T-junctions through linear algebraic analysis. From the subspace in which B-splines exists, we construct a matrix of which the null space is equivalent to the spline space. The shape of domain

is generalized from rectangular to the shape that can be grouped to several rectangles. Furthermore, the regularity imposed on each boundary between neighboring cells even can be n-1 if the degree of the spline space is n. The domain can be extended to one of k-parametric dimensions. From that null space, we get a basis and from the basis we get a partition of unity basis having local support property by multiplying change of basis matrices on it or solving a constrained quadratic model. In addition to this, we give a talk about the level by level local refinement for some specific domain and regularity conditions, especially one of PHT- splines(Polynomial Splines over Hierarchical T-meshes), that guarantees the increment of the dimension whenever the domain is crossly subdivided.

3.26 Sampling conditions and topological guarantees for shape reconstruction algorithms

Andre Lieutier (Dassault Systeme, FR)

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In this talk, I shall first recall some results on shape reconstruction algorithms from sampled points coming with topological guarantees and, for that, recall the definition of some simplicial complexes built on finite set of points in Euclidean space: alpha-shapes, Cech and Rips complexes. Of course, no topological guarantee can be given without associated sampling condition. I shall then gives some properties of the Euclidean distance function to compact sets and associated results for sampling conditions on non smooth sets, mentioning the relation with topological persistence. I shall then quickly introduce sampling conditions and associated results based on the recent notion of defect of convexity.

3.27 Sketching Mesh Cutting

Ligang Liu (Zheiiang University - Hangzhou, CN)

Shape decomposition is an important ingredient in geometry processing and shape analysis. However, it remains a challenge to develop automatic mesh segmentation algorithms due to the complicated human perception. Therefore, user interaction is generally needed to obtain the desired meaningful parts. We will presenting a series of our works on sketch-based interactive shape decomposition. We will illustrate that users can easily express their intentions to obtain the desired semantic subparts by simply drawing some rough sketches over the shapes in an intuitive manner. The users do not care much about how precisely they draw the sketches and the algorithms can obtain the expected decompositions.

3.28 Surface meshing with planar quadrilateral faces

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Yang Liu (Microsoft Research - Beijing, CN)
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Joint work of Yang Liu, Weiwei Xua, Jun Wang, Lifeng Zhu
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Planar quadrilateral (PQ) meshes are essential in architectural geometry for discretizing a freeform architectural structure with planar quad faces due to the economy and easy fabrication. An intuitive tool for designing and generating PQ meshes is much demanded by architects. However the existing methods do not provide a general solution which maximizes degrees of design freedom. To accomplish this task, we present a novel method based on conjugate direction fields (CDF) which allow k/4 ($k \in Z$) singularities. We characterize the smoothness of a CDF by the signed permutation and convert it to a simple summation of trigonometric functions for measurement. Starting with a triangle discretization of a freeform shape, we first compute an as smooth as possible CDF satisfying the user's directional and angular constraints, then apply global parameterization and planarization techniques to generate a PQ mesh which approximates the input shape and follows the CDF well. The effectiveness and robustness of our method are demonstrated on various 3D models.

3.29 Feature Adaptive GPU Rendering of Catmull-Clark Subdivision Surfaces

Charles Loop (Microsoft Research - Redmond, US)

Catmull-Clark subdivision surfaces. Unlike previous methods, our algorithm computes the true limit surface up to machine precision, and is capable of rendering surfaces that conform to the full RenderMan specification for Catmull-Clark surfaces. Specifically, our algorithm can accommodate base meshes consisting of arbitrary valence vertices and faces, and the surface can contain any number and arrangement of semi-sharp creases and hierarchically defined detail. We also present a variant of the algorithm which guarantees watertight positions and normals, meaning that even displaced surfaces can be rendered in a crack-free manner. Finally, we describe a view dependent level-of-detail scheme which adapts to both the depth of subdivision and the patch tessellation density. Though considerably more general, the performance of our algorithm is comparable to the best approximating method, and is considerably faster than Stam's exact method.

3.30 Realistic plant modeling

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Geraldine Morin (ENSEEIHT - IRIT Toulouse, FR)
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Just as in the real world, plants are important objects in virtual worlds for creating pleasant and realistic environments, especially those involving natural scenes. As such, much effort

has been made in realistic modeling of plants. As the trend moves towards networked and distributed virtual environments, however, the current models are inadequate as they are not designed for progressive transmissions. We fill in this gap by proposing a progressive representation for plants based on generalized cylinders. We model the shape and thickness of branches in a plant as Bézier curves, group the curves according to the similarity, and differentially code the curves to represent the plant in a compact and progressive manner. Then, we derive a fully automatic, analysis-by-synthesis method to generate realistic vine plant models from easy-to-get images or video streams.

3.31 Uniformly stable linear 'wavelets' on general triangulations

Knut M. Morken (University of Oslo, NO)

In this talk we construct wavelet-like functions on arbitrary triangulations. Nested triangulations are obtained through refinement by two standard strategies, in which no geometric regularity is enforced. One strategy inserts a new node anywhere inside a triangle and splits it into three smaller triangles by connecting the node to the three vertices of the triangle. The other strategy splits two neighbouring triangles into four smaller triangles by inserting a new node anywhere on the edge between the triangles and connecting this node to the two opposite vertices. The refinement results in nested spaces of piecewise linear functions. These functions are made to satisfy certain orthogonality conditions, which locally correspond to vanishing linear moments. We show that this construction is uniformly stable in the uniform norm, independently of the geometry of the original triangulation and the refinements, and illustrate with an example.

3.32 Rational linear re-parameterizations

Jörg Peters (University of Florida, US)

There is a unique family of rational linear re-parameterizations and associated surface partitions that allow three- or four-sided patches to join with geometric continuity. How can we take advantage of the fact that these re-parameterization preserve the total degree of rational patches?

3.33 CubeCover - Cubical grids for bounded volumes

Konrad Polthier (FU Berlin, DE)

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Joint work of Konrad Polthier, Matthias Nieser, Ulrich Reitebuch

We discuss novel techniques to fill a bounded volumetric shape with a (preferably coarse) cubical voxel structure. Among the optimization goals are alignment of the voxels with the bounding surface as well as simplicity of the voxel grid. Mathematical analysis of the possible singularities is given.

The algorithm is uses a tetrahedral volume mesh plus a user given guiding frame field as input. Then it constructs an atlas of chart functions, i.e. the parameterization function of the volume, such that the images of the coordinate lines align with the given frame field. Formally the function is given by a first-order PDE, namely the gradients of the coordinate functions are the vectors of the frame. In a first step, the algorithm uses a discrete Hodge decomposition to assure local integrability of the frame field. A subsequent step assures global integrability along generators of the first homology group and alignment a face of the boundary cube with the original surface boundary. All steps can be merged into solving linear equations.

Conceptually the presented CubeCover-algorithm extends the known QuadCover-algorithm from surface meshes to volumes meshes.

3.34 Circular Arc Structures

Helmut Pottmann (KAUST - Jeddah, SA)

The most important guiding principle in computational methods for freeform architecture is the balance between cost efficiency on the one hand, and adherence to the design intent on the other. Key issues are the simplicity of supporting and connecting elements as well as repetition of costly parts. We propose so-called circular arc structures as a means to faithfully realize freeform designs without giving up smooth appearance. In contrast to non-smooth meshes with straight edges where geometric complexity is concentrated in the nodes, we stay with smooth surfaces and rather distribute complexity in a uniform way by allowing edges in the shape of circular arcs. We are able to achieve the simplest possible shape of nodes without interfering with known panel optimization algorithms. We study remarkable special cases of circular arc structures which possess simple supporting elements or repetitive edges, we present the first global approximation method for principal patches, and we show an extension to volumetric structures for truly three-dimensional designs.

3.35 The construction(s) of Ck subdivision schemes

Hartmut Prautzsch (Universität Karlsruhe, DE)

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Subdivision algorithms for arbitrary meshes generating Ck surfaces that are also Ck at their extraordinary points have first been described 16 years ago. Several mprovements were developed and published until recently. I show that all possible constructions obey the same simple framework, which is based on the necessary and sufficient Ck conditions for subdivision schemes. I discuss the free parameters in this framework and show their limits and point to parameters that have not been used so far.

3.36 EC funding in the PEOPLE programme – a short tour

Ewald Quak (Technical University - Tallinn, EE)

Based on my experiences as an evaluator for the European framework programmes, I would like to give some pointers concerning funding opportunities, especially in the PEOPLE programme, including also those which concern cooperation with non-European research partners.

3.37 Ambient B-Splines

Ulrich Reif (TU Darmstadt, DE)

We propose a new spline representation of C^k -surfaces of arbitrary topology: To construct such a surface S from given data, say a triangular mesh S', we employ a smooth reference manifold M of equal genus and start with determining a parametrization $\Phi: M \to S'$ of S' over M. Let $v: M \to \mathbb{R}^3$ be a smooth, non-tangential vector field on M, e.g., the Gauss map. Then there exists a neighborhood Ω of M such that any point $y \in \Omega$ can be written as y = x + tv(x) for unique $x \in M$ and $t \in \mathbb{R}$. Now, a trivariate function $F: \Omega \to \mathbb{R}^3$ is constructed by letting

$$F(y) := \Phi(x)$$
.

Then, the function F is approximated by some trivariate tensor product spline $G: \Omega \to \mathbb{R}^3$ of arbitrary degree and smoothness. Finally, G is restricted to M to obtain the desired representation $S: M \to \mathbb{R}^3$ of the surface.

The method is easy to implement and possesses many variants and generalizations. In particular, tensor product B-splines may be replaced by any other set of trivariate basis functions.

3.38 Interpolatory blending net subdivision schemes of Dubuc-Deslauriers type

Lucia Romani (Universita Bicocca - Milan, IT)

In this talk we deal with interpolatory net subdivision schemes, i.e. iterative procedures which repeatedly refine nets of univariate functions and converge to continuous bivariate functions interpolating the initial net. We extend the family of Dubuc-Deslauriers interpolatory point subdivision schemes and construct an interpolatory blending net subdivision scheme. To this purpose, at each recursion step we use a net refinement operator based on the evaluation of an interpolating Gordon surface. Doing so the limit surface is not only interpolating the initial net of univariate functions, but also all the nets generated by the iterative procedure. Convergence and smoothness properties of these blending net subdivision schemes are proved in relation to the properties of the cardinal blending function used to define the Gordon surface, and by exploiting the notion of proximity with the tensor-product Dubuc-Deslauriers schemes for points. We conclude by presenting an example of a family of interpolatory blending net subdivision schemes whose first two members can be used to design C^1 and C^2 surfaces from given nets of 3D curves.

3.39 A superficial survey of approaches to CAD/CAE interfacing

Malcolm A. Sabin (Numerical Geometry Ltd., GB)

The interfacing of CAD models to analysis is a matter of current concern. There are a dozen or so approaches either in use or proposed. The talk outlines these and identifies some issues which cause each of them problems, and then goes on to suggest that we should regard the approaches as sources of good ideas rather than as competing technologies.

3.40 Powell-Sabin B-splines for adaptive local refinement in Isogeometric Analysis

Maria Lucia Sampoli (University of Siena, IT)

In this talk we present a novel approach, within the new paradigm of Isogeometric Analysis introduced by Hughes et al. (2005), to deal with numerical solution of partial differential equations. The new method is based on Powell-Sabin B-splines which, along with the ability of accurately describe the approximate solution, allow also local refinement strategies.

3.41 Creation and Manipulation of Discrete Indicator Functions

Scott Schaefer (Texas A&M University, US)

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Joint work of Scott Schaefer, Josiah Manson, Jason Smith

In this talk, we will explore the creation and extraction of surfaces from Discrete Indicator Functions. These functions are box-filtered versions of exact Indicator functions and have a variety of applications in Computer Graphics and Geometric Modeling. To construct these functions from boundary representations, we will derive exact integrals of a wavelet decomposition of the function. This method yields an efficient algorithm for rasterizing polygons, fonts and implicitizing 3D surfaces. Next we consider the inverse problem of constructing a surface from a discrete indicator function and show that applying Marching Cubes with linear interpolation performs poorly. We provide a simple replacement for linear interpolation that dramatically improves the quality of the results with almost no computational overhead.

3.42 Possible Use of Subdivision Surfaces in Industrial Modelling

Timothy L. Strotman (Siemens – Milford, US)

Industrial modelling consists of two stages, design and analysis. Currently subdivision surfaces are rarely used in the design stage. Many analysis applications, such as finite element modeling, toolpath generation, and visualization, work with different faceted versions of the design part. In this talk we will explore how a single multi-resolution subdivision surface model could replace these facetted models in the analysis applications. We will present the strengths and weaknesses of this approach, hoping to generate a discussion for possible improvements to this approach.

3.43 Smoothing Operation on Spherical Covering of Partition of Unity Implicits

Hiromasa Suzuki (University of Tokyo, JP)

Partition of unity implicit surfaces (PU implicits) is a method proposed in [1] to reconstruct a triangular mesh from a set of scanned points. As with other implicit function based reconstruction approaches, an implicit function is first defined to roughly represent signed distance from the surface underlying the set of points. Then a triangular mesh is extracted as the zero level set of the implicit function by iso-surfacing. In the PU implicits an implicit function is defined by blending linear implicit functions each of which locally approximates the surface within a spherical support. These spherical supports all together form a covering of the set of points.

For dealing with noisy scanned points, it is necessary to apply a smoothing operation on the points or on a reconstructed triangular mesh. We rather derive new smoothing operation acting on the spherical covering of PU implicits [2]. New differential operators are proposed to compose a smoothing operation to the spherical covering. This process achieves noise robust surface reconstruction from scattered points.

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3.44 Iso-geometric Finite Element Analysis Based on Catmull-Clark Subdivision Solids

Georg Umlauf (HTWG Konstanz, DE)

We present a volumetric iso-geometric finite element analysis based on Catmull-Clark solids. This concept allows to use the same representation for the modeling, the physical simulation, and the visualization, optimizing the design process reducing the gap between CAD and CAE. The underlying geometric models are Catmull-Clark surfaces with optional corners and creases. Hence, these models can easily be refined to increase the accuracy of the simulation. The crucial point in the simulation phase is the evaluation at arbitrary parameter values for the assembly of the stiffness matrices. We propose a method similar to the standard subdivision surface evaluation technique, such that numerical quadrature can be used.

Experiments for linear elastic materials show that our approach converges faster than methods based on tri-linear and tri-quadratic elements and is numerically more stable than tri-quadratic finite elements. However, the topological structure of Catmull-Clark elements is as simple as the structure of linear elements. Furthermore, the Catmull-Clark elements we use are C^2 -continuous at the boundary and in the interior, except at extraordinary points.

We also use this technique to solve the incompressible steady-state Navier-Stokes equations for flow simulations. The non-linear system of equations which arises during the simulation is solved by Newton's method. Our experiments show that the approach requires approximately the same number of iterations as linear Lagrangian hexahedral finite elements for the same number of degrees of freedom. As Catmull-Clark finite elements are tri-cubic we can achieve higher accuracy with the same number of degrees of freedom and almost the same computational cost.

3.45 N-sided Transfinite Surface Interpolation Revisited

Tamas Varady (Technical University of Budapest, HU)

Transfinite surface interpolation is a classic topic of CAGD, and various representational schemes to interpolate boundary and cross-derivative functions have been published for non-quadrilateral domains. Discussion starts with Coons patches that can be generalized in three possible ways combining side or corner interpolants. The direct generalization of Coons' scheme apparently has not been explored until now. In order to minimize shape artifacts and to provide a more natural patch interior for uneven and strongly curved boundary configurations former approaches are enhanced and extended. The proposed representation is based on irregular convex domains, special blending functions and balanced sweep-line parameterizations that avoid parametric shearing. Sweep-line constructions to create one- or two-sided patches are also presented.

3.46 Surface fitting with cyclide splines

Wenping Wang (University of Hong Kong, HK)

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Joint work of Wenping Wang, Pengbo Bo and Yang Liu

We consider the problem of fitting a cyclide spline surface to a free-form target surface. It is shown that, by using a global optimization scheme with relaxed corner points and corner frames of cyclide patches, we can fit a free form surface by a G^1 spline surface composed of cyclide patches. T- joints are allowed for adaptive approximation and spherical patches are included in the cyclide spline to fill in singular regions around the umblic points of the underlying target surface.

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