Degradation-Based Maintenance Using Stochastic Filtering for Systems under Imperfect Maintenance

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Outline

Introduction: Main Idea

Solution: On-line Updating

Open Topics & Future Work
Maintenance actions can be classified, according to the efficiency, into 3 types:

- **perfect maintenance**, with each maintenance leaving the system as if it were new;
- **minimal maintenance**, with each maintenance leaving the system in the condition as it was just before the maintenance;
- **imperfect maintenance**, with each maintenance restoring a system’s condition to a younger state but not as good as new.

Compared with the “perfect” and the “minimal” assumptions, it is more realistic that maintenance actions are imperfect.
Intro: main line

How to mathematically quantify the effect of each maintenance?

- **perfect maintenance**: renewal process;
- **minimal maintenance**: non-homogeneous poisson process;
- **imperfect maintenance**: various treatments (Wang 2002).
One of the most popular treatments is to invoke the hazard rate function:

**improvement-factor method**

Let $h(t)$, $t \geq 0$, denote the hazard rate function (**monotonically increasing**) of the target system. Right after a maintenance action at time $t_1 \geq 0$, the hazard rate function changes into $bh(t - t_1 + at_1)$.

- $0 < a < 1$ is an age-reduction factor.
- $b > 1$ is a hazard-rate-increase factor.
If an imperfect maintenance action is taken at time $t_1$, the hazard rate right after the maintenance action changes to $bh(at_1)$.

- There is a decrease in the age of the system ($at_1 < t_1$) and thus a decrease in the hazard rate, indicating that the system becomes younger.
- After the maintenance, the hazard rate increases faster ($b > 1$).

As time elapses, the hazard rate function has the form $bh[(t - t_1) + at_1]$.

- $(t - t_1)$ is the time elapsed from the last maintenance.

Likewise, after the second maintenance at time $t_2$, the hazard rate function has the form

$$b^2 h\{(t - t_2) + a[ (t_2 - t_1) + at_1] \}.$$
A non-stationary Wiener process, \( \{ X_t, \ t \geq 0 \} \), with drift function \( \nu(t) \) and variance parameter \( \sigma^2 \) can be expressed as

\[
X_t = \nu(t) + \sigma B_t.
\]

- \( \nu(t) \) is a monotonically increasing, right-continuous, real-valued function on \( t \geq 0 \) with \( \nu(0) = 0 \).
- \( \{ B_t, \ t \geq 0 \} \) is the standard Brownian motion.

Q

How to mathematically characterize the effect of each imperfect maintenance \textbf{in the context of degradation-based maintenance}?

The improvement-factor method is ineffective since the hazard rate function is extremely complex and non-analytical.
For the non-stationary Wiener process $X_t = \nu(t) + \sigma B_t$, the expected degradation up to time $t$ is $E(X_t) = \nu(t)$.

The first-order derivative of $\nu(t)$, $\nu'(t)$, characterizes the deteriorating speed/rate, termed as degradation rate function.

**lubrication:** The lubricating activity has its impact on the degradation rate function, slowing down the wearing process.

The concept of the improvement factor method can be extended to the degradation rate function $\nu'(t)$ to model maintenance efficiency.
We assume that if an imperfect maintenance action is taken at time $t_1 \geq 0$, the degradation rate function $v'(t)$ after the maintenance has the form $bv'[t - t_1] + a t_1$. Here $0 < a < 1$ is an age-reduction factor, and $b > 1$ is a degradation-rate-increase factor.

- If $a = b = 1$, we arrive at the minimal assumption.
- If $a = 0$ and $b = 1$, we arrive at the perfect assumption.
Two main advantages:

- Deriving the hazard rate function via the first hitting time distribution function is mathematically intractable, especially when the drift function $v(t)$ is non-linear.
- The impact of maintenance actions on the hazard rate is unmeasurable. The impact on the degradation rate can be exteriorized from consecutive degradation measurements.
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An illustrative example: theory

Having built the model to characterize maintenance efficiency, the problem reduces to: **How to evaluate the impact factors a and b?**

Illustrative example:

\[ X_t = \lambda t^\theta + \sigma B_t, \text{ with degradation rate function } \nu'(t) = \lambda \theta t^{(\theta-1)}. \]
An illustrative example: theory

- To simplify matters, we assume that the degradation rate function immediately after a maintenance at time $t$ changes from $v'(t)$ to $bv'(t)$ (no $a$).
- By re-writing
  
  $$bv'(t) = b\lambda \theta t^{(\theta-1)}$$

  as

  $$bv'(t) = \tilde{\lambda} \theta t^{(\theta-1)},$$

  where $\tilde{\lambda} = b\lambda$, we can state that the maintenance activity has its effect on the scale parameter: Right after the maintenance the degradation rate function changes from

  $$v(t) = \lambda \theta t^{(\theta-1)}$$

  to

  $$v(t) = \tilde{\lambda} \theta t^{(\theta-1)}.$$
To quantify the effectiveness of each maintenance action, we need to estimate the value of the improvement factor $b$ after each maintenance.

To estimate the value of the improvement factor $b$ after each maintenance, it is equivalent just to assess the value of the scale parameter $\lambda$ after each maintenance.
Degradation-measuring points are equally spaced with $\Delta > 0$ being the constant time between two consecutive degradation-measuring points, i.e., measuring at epochs $i \Delta$ ($i = 1, 2, 3, ...$).
At each degradation-measuring point we

- first, measure the degradation of the maintaining system;
- second, assess the value of the scale parameter based on the history of maintenance actions and degradation measurements;
- third, decide whether or not to take maintenance action (I will not talk).
An illustrative example: theory

- Denote $x_i$ to be the degradation measurement at time $i \triangle$ with $x_0 = 0$.
- Denote $\lambda_i$ to be the hidden true value of the scale parameter at time $i \triangle$ before any maintenance action.

By assuming that the improvement factor $b$ is a random variable following the normal distribution $N(\bar{b}, Q)$, we have process equation

$$\lambda_i = b_{i-1}\lambda_{i-1} + w_{i-1}, \quad (3.1)$$

and measurement equation

$$x_i - x_{i-1} = \eta_i\lambda_i + \omega_i, \quad (3.2)$$

in which $b_i$ and $\eta_i$ are known coefficients; the process noise $w_i$ and measurement noise $\omega_i$ are both Gaussian white.
An illustrative example: theory

Eq. (3.1) and Eq. (3.2) construct a Kalman filter. At the time $i \triangle$, given the new measurement $x_i$, we define

- $\hat{\lambda}_i$ to be the a posterior estimate of the true value $\lambda_i$,
- $P_i$ to be the a posterior error variance of the estimate $\hat{\lambda}_i$.

Hence, to recursively estimate the scale parameter, the Kalman filter the following updating equations:

$$
\hat{\lambda}_i = b_{i-1}\hat{\lambda}_{i-1} + \frac{(b_{i-1}^2P_{i-1} + Q_{i-1})\eta_i}{(b_{i-1}^2P_{i-1} + Q_{i-1})\eta_i^2 + \sigma^2 \triangle} \times \left( y_i - \eta_i b_{i-1}\hat{\lambda}_{i-1} \right),
$$

$$
P_i = \frac{(b_{i-1}^2P_{i-1} + Q_{i-1}) \times \sigma^2 \triangle}{(b_{i-1}^2P_{i-1} + Q_{i-1})\eta_i^2 + \sigma^2 \triangle}.
$$

(3.4)
An illustrative example: theory

Summation:

- To evaluate the effect of the \((i - 1)\)th maintenance, we only need to assess, at time \(i \triangle\), the value of the scale parameter \(\lambda_i\).
- \(\hat{\lambda}_i\) is the estimate on the scale parameter \(\lambda_i\), and \(P_i\) characterizes the accuracy of the estimate \(\hat{\lambda}_i\).
- \(\hat{\lambda}_i\) and \(P_i\) can be easily obtained by using equations (3.3) and (3.4).

Problem is solved!
Degradation-measuring points are equally spaced with \( \Delta > 0 \) being the constant time between two consecutive degradation-measuring points, i.e., measuring at epochs \( i \Delta \) (\( i = 1, 2, 3, ... \)).
An illustrative example: simulated data \( \{x_1, x_2, \ldots, x_i, \ldots\} \)

At the first point \( \Delta \), simulate \( x_1 \sim \mathcal{N}(\lambda \Delta^\theta, \sigma^2 \Delta) \).

- If a maintenance activity is then taken at epoch \( \Delta \), the value of the scale parameter immediately after the maintenance changes to \( b_1 \lambda \), with \( b_1 \sim \mathcal{N}(\bar{b}, Q) \). Therefore, at the second point \( 2 \Delta \), simulate a degradation increment
  \[
  y_2 \sim \mathcal{N}\left(b_1 \lambda (2^\theta - 1) \Delta^\theta, \sigma^2 \Delta\right).
  \]

- If no maintenance is taken at epoch \( \Delta \), at the second point simulate a degradation increment
  \[
  y_2 \sim \mathcal{N}\left(\lambda (2^\theta - 1) \Delta^\theta, \sigma^2 \Delta\right).
  \]

The second observation of degradation is hence \( x_2 = x_1 + y_2 \).
An illustrative example: simulated data \( \{x_1, x_2, \ldots, x_i, \ldots\} \)

By analogy, at the \( i \)th point, \( i = 2, 3, 4, \ldots \), simulate a degradation increment

\[
y_i \sim N \left( \prod_{j=1}^{i-1} b_j^{m_j} \lambda \left[ i^\theta - (i - 1)^\theta \right] \Delta^\theta, \sigma^2 \Delta \right),
\]

and the \( i \)th degradation measurement is set to be \( x_i = x_{i-1} + y_i \). Here \( m_i \) \( (i = 1, 2, 3, \ldots) \) are the indicators with \( m_i = 1 \) indicating that the system is maintained at the \( i \)th point.
An illustrative example: results

Apply the proposed stochastic filter on the simulated data \( \{x_1, x_2, ..., x_i, ...\} \).

Since the underlying true value of the scale parameter at each point is known, we calculate the value of the a posterior error variance \( P_i \) and the value of the deviation \( \hat{\lambda}_i - \lambda_i \).

The evolutions of \( \{\hat{\lambda}_i - \lambda_i, \ i = 1, 2, 3...\} \) and \( \{P_i, \ i = 1, 2, 3...\} \) are depicted in the following figure.
An illustrative example: results

The deviation and the error variance converge to zero rapidly, showing the efficiency of the proposed algorithm.
An illustrative example: results

To show the robustness of the algorithm, we run the simulation for 100 times. The 100 evolving paths of the deviation $\hat{\lambda}_i - \lambda_i$ and 100 evolving paths of the error variance $P_i$ are plotted in the following figure.
An illustrative example: results

Figure: Evolution paths of \( \{ \hat{\lambda}_i - \lambda_i, \ i = 1, 2, 3... \} \) and \( \{ P_i, \ i = 1, 2, 3... \} \) with 100 samples.

All the deviations and variances converge rapidly to zero, showing the robustness of the stochastic filtering algorithm.
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Open topics

Future research can be carried out in

- implementing this strategy into various condition-based maintenance schemes,
- studying the case in which both the age-reduction factor and the degradation-rate-increase factor are involved,
- dealing with other degradation processes via stochastic filtering technique.