Imperfect Repair Systems: Test and Model Selection

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Outline

• Motivating Situation: Dump Trucks Data
• Counting Process Formulation
• Non-parametric Test for Imperfect Repair
• ARA and ARI Classes of IR Models
• Models Selection/Reliability Prediction
• Dump Trucks Data Revisited
• Final Remarks
Motivating situation

Mining DUMP truck
Motivating situation

✓ Designed to operate in road conditions.

✓ DUMP trucks are used to transport mining production.

✓ In the mine under study, they are used in much more severe conditions.

✓ Engineering interest: engine failures.
Motivating Situation (cont.)

DATA SET

✓ Data collection in the period: July to October, 2012, for 5 trucks;

✓ The accumulated number of working days were registered;

✓ A total of 129 failures were observed, each one followed by a repair;
Motivating situation (cont.)

Failure times in days of operation ("o" are failures).
Motivating situation (cont.)

Empirical Mean Cumulative Function (MCF)
OBJECTIVES

✓ Estimate the effect of the repairs and the aging speed of the engine’s trucks.

✓ Estimate reliability predictors to provide information in order to base the decision-making process related to maintenance policies.
Motivating situation (cont)

Specific Objectives

✓ Is the repair imperfect or minimum?

✓ Reliability Prediction:
  • Model selection.
  • Estimate repair and age effects and reliability Predictors.
General modeling of a counting process

- $N(t)$: number of observed failures up to time $t$;

- $\{N(t)\}_{t \geq 0}$: counting processes that is characterized by the failure intensity function (Andersen et.al, 1993):

$$
\lambda(t) = \lim_{h \to 0} \frac{P(N(t + h) - N(t) = 1|\mathcal{G}_t)}{h}, \quad \forall t \geq 0
$$

where $\mathcal{G}_t$ is the history of the process up to time $t$ ($T_1 = t_1, \ldots, T_{N(t)} = t_{N(t)}$).
General modeling of a counting process (cont.)

- Cumulative Intensity: $\Lambda(t) = \int_0^t \lambda(u) du$.

- Mean Cumulative Intensity: $\Phi(t) = E(N(t)) = E[\Lambda(t)]$, (Aalen, 1978).

- ROCOF (rate of occurrence of failures) function is

  $$\phi(t) = \Phi'(t) = E[\lambda(t)]$$
Repair Types

1. Perfect Repair (PR)
   - system leaves as it were new (As Good As New).
   - Renew Process.

2. Minimal Repair (MR)
   - system stays in the same condition as before the failure (As Bad As Old).
   - NHPP: $\phi(t) = \lambda(t)$.

3. Imperfect Repair
   - system leaves in a condition between ABAO and AGAN.
   - Probably the one appropriate for the Mining Dump Truck, according to the maintenance engineers.
Non-Parametric Test for Minimum Repair

\[ H_0 : \text{MR} \quad \text{vs} \quad H_1 : \text{Non Harmful Repair/First Order} \]

It means:

- Non Harmful Repair: System reliability improves after repair.
- First Order: \( \lambda(t) \) just depend on the last failure, \( t_N(t) \).
• Under $H_0$: MR, systems are a homogeneous NHPPs sample, with an increasing $\lambda(t)$ in $t$.
• Under $H_0$: MR, each system would have the same probability to be the next failure’s system.
• Under $H_1$: no harmful/first order repair, failure system will take longer time for the next failure as compared with the others.
Notation and Assumptions

- $k$ identical repairable systems, where the failures history occurs independently;

- at each failure, a repair action of negligible length is performed;

- $n_i$ failures are observed in the $i^{th}$ system, $i = 1, 2, \ldots, k$;

- $N = \sum_{i=1}^{k} n_i$ is the total number of observed failures in the systems.

- Let $r_l; l = 1, \ldots, N$, be the rank of the observed failure times in the overall sample; $r_1 < r_2 < \ldots < r_N$.

- $Z_{i,l} = 1$ ($G_l = i$), for the systems membership.
Non-Parametric Test for Minimum Repair

- Let’s define,

\[ X_l = \begin{cases} 
1, & Z_i,l = Z_i,l+1, \text{ for all } i = 1, \ldots, k \\
0, & \text{otherwise}
\end{cases} \]

for \( l = 1, \ldots, N - 1 \).

- Let’s define the test statistic: \( T = \sum_{l=1}^{N-1} X_l \).
Non-Parametric Test Idea

\[ X_1 = X_2 = X_3 = X_5 = X_6 = X_8 = 0 \text{ and } X_4 = X_7 = 1 \text{ and } T = 2. \]
Non-Parametric Test

• Therefore, under $H_0$, $T$ has a bin($N - 1, \pi = 1/k$).

• Test can be reformulated in the following terms:

$$H_0 : \pi = \frac{1}{k} \text{ vs } H_1 : \pi < \frac{1}{k}.$$  

• For an observed $T=t$, p-value = $P(T \leq t | \pi = 1/k)$. 
Non-Parametric Test for Minimum Repair

Small size Monte Carlo Simulation

- Descriptive size evaluation of the test

- Scenarios
  - Number of systems: \( K = 5, 10 \);
  - Truncation times: \( T = 5, 10, 15 \);
  - Power Law Process

\[
\lambda(t) = \frac{\beta}{\eta} \left\{ \frac{t}{\eta} \right\}^{\beta-1}
\]

- \( \eta = 1 \) and \( \beta = 1.5, 2, \)
- 10000 replicates.
Non-Parametric Test for Imperfect Repair

Tabela: Monte Carlo Simulation Results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Coverage</th>
<th>( \hat{\Phi}/\text{System} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = 5 )</td>
<td>( T = 5 )</td>
<td>( \beta = 1.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta = 2 )</td>
</tr>
<tr>
<td></td>
<td>( T = 10 )</td>
<td>( \beta = 1.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta = 2 )</td>
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<tr>
<td></td>
<td>( T = 15 )</td>
<td>( \beta = 1.5 )</td>
</tr>
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<td></td>
<td></td>
<td>( \beta = 2 )</td>
</tr>
<tr>
<td>( K = 10 )</td>
<td>( T = 5 )</td>
<td>( \beta = 1.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta = 2 )</td>
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<tr>
<td></td>
<td>( T = 10 )</td>
<td>( \beta = 1.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta = 2 )</td>
</tr>
<tr>
<td></td>
<td>( T = 15 )</td>
<td>( \beta = 1.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta = 2 )</td>
</tr>
</tbody>
</table>

- Dump trucks data: p-value = 0.08544. It is an indication against the hypothesis of minimum repair.
Imperfect Repair (IR) Models

  - Virtual age models: rejuvenate the system after repair;
  - the degree of efficiency of the repair is represented by $\theta$ ($0 \leq \theta \leq 1$).

- Doyen and Gaudoin (2004): ARA and ARI classes of models.
  - start at $t = 0$ with an initial intensity $\lambda_R(t)$.
  - has a rule to define how intensity changes immediately after a failure/repair.
ARA class of models (Doyen and Gaudoin, 2004)

- ARA$_m$ model: repair reduces the increment in system age since the last $m$ failures (memory parameter). Its failure intensity function is expressed by

$$\lambda_{ARA_m}(t) = \lambda_R(t - (1 - \theta) \sum_{j=0}^{\text{Min}(m-1,N(t)-1)} \theta^j T_{N(t)-j}).$$

- Virtual age model proposed by Kijima et al. (1988) corresponds to the ARA$_1$ ($m = 1$).
Repair action reduces the failure intensity function of the system;

repair reduces the increment in failure intensity since the last $m$ failures;

Its failure intensity function is expressed by:

$$
\lambda_{ARI_m}(t) = \lambda_R(t) - (1 - \theta) \sum_{j=0}^{\text{Min}(m-1,N(t)-1)} \theta^j \lambda_R(T_{N(t)-j}).
$$
Likelihood Function for a Counting Processes

- a counting process $N(t)$ observed from 0 until a time $T$, $0 = t_0 < t_1 < \ldots < t_{N(T)} \leq t^*$


- ARA and ARI models.
Likelihood function for the ARA class of models

\[ L_{ARA_m}(\mu) = \]

\[ = \prod_{i=1}^{k} \prod_{j=1}^{n_i} \{ \lambda_R(t_{i,j} - (1 - \theta) \sum_{p=0}^{\min(m-1,j-2)} \theta^p t_{i,j-1-p}) \times \]

\[ \times e^{-\Lambda_R(t_{i,j} - (1 - \theta) \sum_{p=0}^{\min(m-1,j-2)} \theta^p t_{i,j-1-p}) + \Lambda_R(t_{i,j-1} - (1 - \theta) \sum_{p=0}^{\min(m-1,j-2)} \theta^p t_{i,j-1-p})} \times \]

\[ \times e^{-\Lambda_R(t_{i}^* - (1 - \theta) \sum_{p=0}^{\min(m-1,n_i-1)} \theta^p t_{i,n_i-p}) + \Lambda_R(t_{i,n_i} - (1 - \theta) \sum_{p=0}^{\min(m-1,n_i-1)} \theta^p t_{i,n_i-p})} \]
Likelihood function for the ARI class of models

\[ L_{\text{ARI}_m}(\mu) = \]

\[ = \prod_{i=1}^{k} \prod_{j=1}^{n_i} \left\{ \left[ \lambda_R(t_{i,j}) - (1 - \theta) \sum_{p=0}^{\min(m-1,j-2)} \theta^p \lambda_R(t_{i,j-1-p}) \right] \times \right. \]

\[ \times e^{-\Lambda_R(t_{i,j}) + \Lambda_R(t_{i,j-1}) + (1-\theta)[t_{i,j} - t_{i,j-1}] \sum_{p=0}^{\min(m-1,j-2)} \theta^p \lambda_R(t_{i,j-1-p})} \right\} \times \]

\[ \times e^{-\Lambda_R(t_{i}^*) + \Lambda_R(t_{i,n_i}) + (1-\theta)[t_{i}^* - t_{i,n_i}] \sum_{p=0}^{\min(m-1,n_i-1)} \theta^p \lambda_R(t_{i,n_i-p})}, \]
Parameter Estimation in ARA and ARI models

For both cases, the likelihood function was rewritten assuming a PLP parametric form for the initial intensity:

\[ \lambda_R(t) = \frac{\beta}{\eta} \left\{ \frac{t}{\eta} \right\}^{\beta-1} \]

therefore, \( \mu = (\beta; \eta; \theta) \) is the vector of parameters to be estimated.
Model selection

• maximum value of the estimated likelihoods: \( \hat{L} = L(\hat{\theta}; \hat{\beta}; \hat{\eta}) \)

• Burnham and Anderson (2004) : weight of evidence in favor of model \( r \), given by:

\[
w_r = \frac{\exp(-\Delta_r/2)}{\sum_{r=1}^{R} \exp(-\Delta_r/2)}
\]

where

• \( \Delta_r = \hat{L}_{max} - \hat{L}_r, \ (r = 1, \ldots, R) \)

• and \( \hat{L}_{max} \) is the maximum of the \( R \) different \( \hat{L} \) values, considering that \( R \) different models were fitted.

This transformation forces the best model to have \( \Delta = 0 \), while the rest of the models have positive values.
Model selection - Goodness-of-fit plot

- MCF $\Phi(.)$ is estimated to each model according to these steps:
  - ML estimates are obtained for a model from its observed failure data.
  - Observed failure data for the $i$-th system ($i = 1, \ldots, k$) and the MLEs are plugged in the model intensity function, providing $\hat{\lambda}_i(t)$.
  - $\hat{\Phi}_i(t)$ is computed as $\int_0^t \hat{\lambda}_i(u)du$, for $0 \leq t \leq t_i^*$.
  - Finally, $\hat{\Phi}(t)$ is obtained as $\frac{\sum_{i=1}^K \hat{\Phi}_i(t)}{K}$.

- Plot of $\hat{\Lambda}(t)$ against empirical MCF (Nelson-Aalen plot).
The following models were considered:

- $ARA_m; \ m = 1, \ldots, 31$
- $ARI_m; \ m = 1, \ldots, 31$
- Minimal Repair ($\theta = 1$)

In cases 1) and 2), $m = 31$ corresponds to $m = \infty$ (max number of failures) and the PLP parametric form for the initial intensity was adopted.
## Tabela: Results of the model fitting - Dump trucks data

<table>
<thead>
<tr>
<th>Estimated values</th>
<th>MODELS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MR</td>
<td>$ARA_1$</td>
<td>$ARA_{13}$</td>
<td>$ARA_{\infty}$</td>
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<tr>
<td>$\hat{\beta}$</td>
<td>1.14</td>
<td>1.33</td>
<td>1.80</td>
<td>1.81</td>
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<tr>
<td></td>
<td>[0.96;1.35]</td>
<td>[1.05;1.69]</td>
<td>[1.39;2.35]</td>
<td>[1.39;2.35]</td>
</tr>
<tr>
<td>$\hat{\eta}$</td>
<td>5.92</td>
<td>4.94</td>
<td>7.58</td>
<td>7.59</td>
</tr>
<tr>
<td></td>
<td>[3.54;9.93]</td>
<td>[3.41;7.15]</td>
<td>[5.35;10.76]</td>
<td>[5.35;10.79]</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>-</td>
<td>0.02</td>
<td>0.60</td>
<td>0.60</td>
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<td>-</td>
<td>[0.0001;0.53]</td>
<td>[0.43;0.84]</td>
<td>[0.42;0.84]</td>
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<tr>
<td>$\hat{L}$</td>
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<td>-304.7039</td>
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<td></td>
<td>$ARI_1$</td>
<td>$ARI_{13}$</td>
<td>$ARI_{\infty}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>1.42</td>
<td>1.89</td>
<td>1.90</td>
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<td></td>
<td>[1.06;1.91]</td>
<td>[1.69;2.11]</td>
<td>[1.71;2.11]</td>
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<tr>
<td>$\hat{\eta}$</td>
<td>4.18</td>
<td>7.48</td>
<td>7.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.57;6.79]</td>
<td>[5.74;10.11]</td>
<td>[5.76;10.17]</td>
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</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>0.23</td>
<td>0.67</td>
<td>0.67</td>
<td></td>
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<tr>
<td></td>
<td>[0.05;1.00]</td>
<td>[0.52;0.87]</td>
<td>[0.52;0.87]</td>
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<tr>
<td>$\hat{L}$</td>
<td>-306.2146</td>
<td>-300.0904</td>
<td>-300.1155</td>
<td></td>
</tr>
</tbody>
</table>
Figura: Criterion value for model selection (0 - MR, + ARI and Δ - ARA).
Dump trucks data revisited (cont.) - GOF Plots
for $ARI_\infty$, $\hat{\beta} = 1.90$ ([1.71; 2.11]), indicating that the equipment failure intensity function increases with time (intrinsic aging)

$\hat{\theta} = 0.67$ ([0.52,0.87]). The repairs after failures tend to leave the equipment in a state between AGAN and ABAO.
Reliability Prediction Functions - \((ARIL_m\) class of models).

\[
R_{T_n,ARIL_m}(t) = P(T_{n+1} - T_n > t|\mathcal{G}_t) = P(N(T_n, T_n + t] = 0|\mathcal{G}_t) = \exp \left\{ \left( \frac{t_n}{\eta} \right)^\beta - \left( \frac{t_n + t}{\eta} \right)^\beta \right\} \times \\
\times \exp \left\{ t(1 - \theta) \sum_{j=0}^{\text{Min}(m-1,n)-1} \theta^j \frac{\beta}{\eta} \left( \frac{t_{n-j}}{\eta} \right)^{\beta-1} \right\}.
\]
Figura: Estimated reliability functions at $T=106.429$ days for the trucks data set under the fitted model.
Final Remarks

✓ A Non-Parametric test for MR was proposed against the IR alternative.

✓ A extension of the test and a Graphical Technique are still under development.

✓ Imperfect Repair Classes of Models (ARA and ARI): Inference, models Selection and Reliability Predictors).

✓ Illustration in a real data set related to mining dump trucks.

✓ Effect of repair, aging speed and reliability predictor for maintenance policy were estimated for the dump trucks data.
ARA and ARI imperfect repair models: estimation, goodness-of-fit and reliability prediction

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